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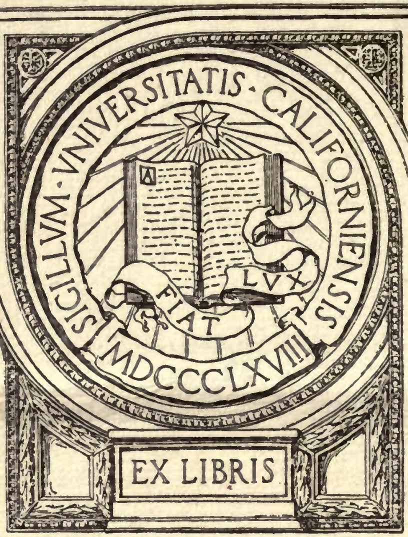
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THE ELEMENTS

OF

ASTRONOMY:

DESIGNED FOR THE USE OF STUDENTS IN
THE UNIVERSITY.

BY THE

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A

SYSTEM

OF

ASTRONOMY.

CHAPTER I.

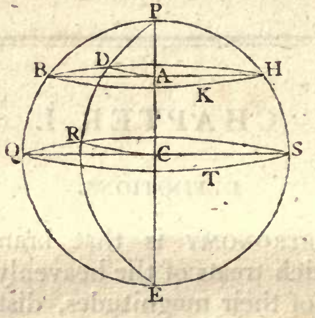
DEFINITIONS.

Art. 1. Astronomy is that branch of Natural Philosophy which treats of the heavenly bodies. The determination of their magnitudes, distances, and the orbits which they describe, is called *plane* or *pure* Astronomy; and the investigation of the causes of their motions is called *physical* Astronomy. The *former* discoveries are made from observations on their apparent magnitudes and motions; and the *latter* from analogy, by applying those principles and laws of motion by which bodies on and near the earth are governed, to the other bodies in the system. The principles of plane Astronomy only are what we here propose to treat of, and we shall begin with the explanation of such terms as are the foundation of the science.

(2.) A *great* circle $QRST$ of a sphere is one whose plane passes through it's center C; and a *small* circle $BDHK$ is that whose plane does not pass through it's center.

(3.) A diameter PCE of a sphere, perpendicular to any great circle $QRST$, is called the *axis* of that great circle; and the extremities P, E, of the axis, are called it's *Poles*.

(4.) Hence, the pole of a great circle is 90° from every point of it upon the sphere; because every angle PCR being a right angle, the arc PR is every where 90°. And as the axis PE is perpendicular to the circle $QRST$ when it is perpendicular to any two radii CQ, CR, a point on the surface of the sphere 90° distant from two points of a great circle wherever taken, will be the pole.



(5.) All angular distances on the surface of a sphere, to an eye at the center, are measured by the arcs of *great* circles.

(6.) Hence, all the triangles formed on the surface of a sphere, for the solution of spherical problems, must be formed by the arcs of *great* circles.

(7.) Any two great circles bisect each other; for both passing through the center of the sphere, their common section must be a diameter of each, and every diameter bisects a circle.

(8.) *Secondaries* to a great circle, are great circles which pass through it's poles; thus, PRE is a secondary to $QRST$.

(9.) Hence, secondaries must be perpendicular to their great circles; for if one line be perpendicular to a plane, any plane passing through that line will also be perpendicular to it; therefore as the axis *PE* of the great circle *QRST* is perpendicular to it, and is the common diameter of all the secondaries, they must all be perpendicular to the great circle. Hence also, every secondary, bisecting it's great circle (*7), must bisect every small circle *BDHK* parallel to it; for the plane of the secondary passes not only through the center *O* of the great circle, but also the center *A* of the small circle parallel to it.

(10.) Hence, a great circle passing through the poles of two great circles, must be perpendicular to each; and, vice versâ, a great circle perpendicular to two other great circles must pass through their poles.

(11.) If an eye be in the plane of a circle, that circle appears a straight line; hence, in the representation of the surface of a sphere upon a plane, those circles whose planes pass through the eye, are represented by straight lines.

(12.) The angle formed by the circumferences of two great circles on the surface of a sphere, is equal to the angle formed by the planes of those circles; and is measured by the arc of a great circle intercepted between them, and which is a secondary to each.

For let *C* be the center of the sphere, *PQE*, *PRE* two great circles; then as the circumferences of these circles at *P* are perpendicular to the common intersection *PCE*, the angle at *P* between them is equal to the angle between the planes, by Euc. B. XI. Def. 6. Now draw *CQ*, *CR* perpendicular to *PCE*; then as these lines are respectively parallel to the directions of the circumferences *PQ*, *PR*, at the point *P*, the angle *QCR* is equal to the angle at *P* formed by the two circles, Euc. B. XI. Prop. 10.; and the angle *QCR* is measured by the arc *QR* of a great circle whose pole is *P*, because *PQ*, *PR* are each 90°.

---

* The figures in the parentheses refer to the Articles.

(13.) If at the intersection P of two great circles as a pole, a great circle $QRST$ be described, and also a small circle $BDHK$ parallel to it, the arcs QR, BD of the great and small circles intercepted between the two great circles, contain the same number of degrees.

For C and A are the centers of the respective circles, and QC is parallel to BA, and RC is parallel to DA; therefore by Euc. B. XI. Prop. 10. the angle BAD is equal to the angle QCR, consequently the arcs BD, QR contain the same number of degrees.

Hence, the arc BD of such a small circle measures the angle at the pole between the two great circles. Also, $QR : BD :: QC : BA ::$ radius : cos. BQ.

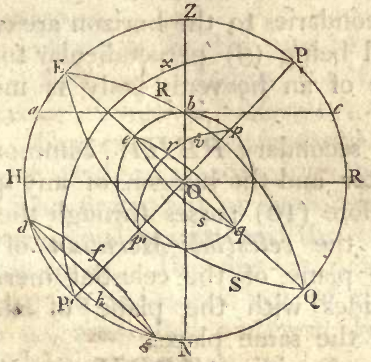
(14.) The *axis* of the earth $pep'q$ is that diameter pOp' about which it performs it's diurnal rotation; and the extremities p, p', of this diameter, are called it's *Poles*.

(15.) The *terrestrial Equator* is a great circle $erqs$ of the earth perpendicular to it's axis. Hence, the axis and poles of the earth are the axis and poles of it's equator. That half of the earth (suppose epq) which lies on the side of the equator which we inhabit, is called the *northern hemisphere*, and the other $ep'q$ the *southern*; and the poles are respectively called the *north* and *south* poles.

(16.) The *Latitude* of a place on the earth's surface, is it's angular distance from the equator, measured upon a secondary to it; thus, the arc eb measures the latitude of b. The secondaries to the equator are called *Meridians*.

(17.) The *Longitude* of a place on the earth's surface, is an arc of the equator intercepted between the meridian passing through the place, and another, called the *first* meridian, passing through that place from which you begin to measure; thus, the longitude of the place v on the meridian prp' measured from the first meridian pep', is er.

(18.) If the plane of the *terrestrial* equator $erqs$ be produced to the sphere of the fixed stars, it marks out a circle $ERQS$ called the *celestial* equator; and if the axis of the earth pOp' be produced in like manner, the points P, P', in the Heavens to which it is produced, are called *Poles*, being the poles of the celestial equator. The star nearest to each pole is called the *Pole star*.



(19.) Secondaries, as PxP', to the celestial equator are called *Circles of Declination*; of these, 24 which divide the equator into equal parts, each containing 15°, are called *hour* circles.

(20.) Small circles, as $dfgh$, parallel to the celestial equator, are called *Parallels of Declination.*

(21.) The *sensible* horizon is that circle abc in the heavens whose plane touches the earth at the spectator b. The *rational* horizon is a great circle HOR in the heavens, passing through the earth's center, parallel to the sensible horizon.

(22.) *Almacanter* is a small circle parallel to the horizon.

(23.) If the radius Ob of the earth at the place b where the spectator stands, be produced both ways to the heavens, that point Z vertical to him is called the *Zenith*, and the opposite point N the *Nadir*. Hence, the *zenith* and *nadir* are the poles of the *rational* horizon (3); for the radius produced being perpendicular to the sensible, must also be perpendicular to the rational horizon.

(24.) Secondaries to the horizon are called *vertical* circles; and being (9) perpendicular to the horizon, the altitude of an heavenly body is measured upon them.

(25.) A secondary *PEHP′* common to the celestial equator and the horizon of any place *b*, and which therefore (10) passes through the poles *P, Z,* of each, is the *celestial Meridian* of that place. Hence, the plane of the celestial meridian of any place coincides with the plane of the terrestrial meridian of the same place.

(26.) The meridian *ZPRH* cuts the horizon in the point *R,* called the *north* point, and in the point *H,* called the *south* point; *P* being the north pole.

(27.) The meridian of any place divides the heavens into two hemispheres lying to the east and west; that lying to the east is called the *eastern* hemisphere, and the other, lying to the west, is called the *western* hemisphere.

(28.) The vertical circle which cuts the meridian of any place at right angles is called the *prime* vertical; and the points where it cuts the horizon are called the *east* and *west* points. Hence, the east and west points are 90° distant from the north and south points. These four are called the *cardinal* points.

(29.) If a body be referred to the horizon by a secondary to it, the distance of that point of the horizon from the north or south points is called it's *Azimuth.* The *Amplitude* is the distance from the east or west point.

(30.) The *Ecliptic* is that great circle in the heavens which the sun appears to describe in the course of a year.

(31.) The ecliptic and equator being great circles must (7) bisect each other, and their inclination is called the *obliquity of the ecliptic*; also, the points where they intersect are called the *equinoctial* points. The times when the sun comes to these points are called the *Equinoxes*.

(32.) The ecliptic is divided into 12 equal parts, called *Signs*: Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricornus ♑, Aquarius ♒, Pisces ♓. The order of these is according to the motion of the sun. The first point of Aries coincides with one of the equinoctial points, and the first point of Libra with the other. The first six signs are called *northern*, lying on the *north* side of the equator; and the last six are called *southern*, lying on the *south* side. The signs ♑, ♒, ♓, ♈, ♉, ♊, are called *ascending*, the sun approaching our (or the north) pole whilst it passes through them; and ♋, ♌, ♍, ♎, ♏, ♐, are called *descending*, the sun receding from our pole as it moves through them.

(33.) When the motion of the heavenly bodies is according to the order of the signs, it is called *direct*, or *in consequentia*; and when the motion is in the contrary direction, it is called *retrograde*, or *in antecedentia*. The *real* motion of all the planets is according to the order of the signs, but their *apparent* motion is sometimes in an opposite direction.

(34.) The *Zodiac* is a space extending on each side of the ecliptic, within which the motions of all planets are performed.

(35.) The *right ascension* of a body is an arc of the equator intercepted between the first point of aries and a declination circle passing through the body, measured according to the order of the signs.

(36.) The *oblique ascension* is an arc of the equator intercepted between the first point of Aries and that point of the equator which rises with any body, measured according to the order of the signs.

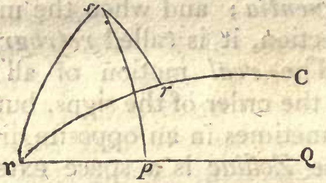
(37.) The *ascensional difference* is the difference between the right and oblique ascension.

(38.) The *Declination* of a body is it's angular distance from the equator, measured upon a secondary to the equator drawn through the body.

(39.) The *Longitude* of a star is an arc of the ecliptic intercepted between the first point of Aries and a secondary to the ecliptic passing through the star, measured according to the order of the signs. If the body be in our system, and seen from the *sun*, it is called the *heliocentric* longitude; but if seen from the *earth*, it is called the *geocentric* longitude; the body in each case being referred perpendicularly to the ecliptic in a plane passing through the eye.

(40.) The *Latitude* of a star is it's angular distance from the ecliptic, measured upon a secondary to the ecliptic drawn through the star. If the body be in our system, it's angular distance from the ecliptic seen from the *earth* is called the *geocentric* latitude; but if seen from the *sun*, it is called the *heliocentric* latitude.

(41.) Thus, if ♈Q be the equator, ♈C the ecliptic, ♈ the first point of Aries, s a star, and the



great circles sr, sp be drawn perpendicular to ♈C and ♈Q; then ♈p is it's right ascension, sp it's declination, sr it's latitude, and ♈r it's longitude. The circle sr is called a *Circle of Latitude*.

Hènce, if we know the right ascension ♈p, and declination ps of a body s, we know it's place; for, take ♈p = to the given right ascension, draw the meridian ps, and take ps = to the given declination, and s is the place of the body. In like manner, if we know the longitude ♈r of a body s, and latitude rs, we know the place of the body.

(42.) The *Tropics* are two parallels of declination touching the ecliptic. One, touching it at the beginning of Cancer, is called the *Tropic of Cancer;* and the other, touching it at the beginning of Capricorn, is called the *Tropic of Capricorn.* The two points where the tropics touch the ecliptic, are called the *Solstitial* points.

(43.) The *Colures* are two secondaries to the celestial equator; one, passing through the equinoctial points, is called the *equinoctial* colure; and the other, passing through the solstitial points, is called the *solstitial* colure. The times when the sun comes to the solstitial points are called the *Solstices.*

(44.) The *Arctic* and *Antarctic* circles are two parallels of declination, the former about the north and the latter about the south pole, the distances of which from the two poles are equal to the distances of the tropics from the equator. These are also called *polar* circles.

(45.) The two tropics, and two polar circles, when referred to the earth, divide it into five parts, called *Zones;* the two parts within the polar circles are called the *frigid* zones; the two parts between the polar circles and the tropics are called the *temperate* zones; and the part between the tropics is called the *torrid* zone. Small circles in the heavens are referred to the earth, and the contrary, by lines drawn to the earth's center.

(46.) A body is in *Conjunction* with the sun, when it has the same longitude; in *Opposition,* when the difference of their longitudes is 180°; and in *Quadrature,* when the difference of their longitudes is 90°. The conjunction is marked thus ☌, the opposition thus ☍, and quadrature thus □.

(47.) *Syzygy* is either conjunction or opposition.

(48.) The *Elongation* of a body from the sun, is it's angular distance from the sun when seen from the earth.

(49.) The *diurnal parallax* is the difference between the apparent places of a body in our system when referred to the fixed stars, if seen from the center and surface of the earth. The *annual* parallax is the difference between the apparent places of a body in the heavens, when seen from the opposite points of the earth's orbit.

(50.) The *Argument* is a term used to denote any quantity by which another required quantity may be found. For example, the argument of a planet's latitude is it's distance from it's node, because upon that the latitude depends.

(51.) The *Nodes* are the points where the orbits of the primary planets cut the ecliptic, and where the orbits of the secondaries cut the orbits of their primaries. That node is called *ascending* where the planet passes from the south to the north side of the ecliptic; and the other is called the *descending* node. The ascending node is marked thus ☊, and the descending node thus ☋. The line which joins the nodes is called the *line of the nodes.*

(52.) If a perpendicular be drawn from a planet to the ecliptic, the angle at the sun between two lines, one drawn from it to that point where the perpendicular falls, and the other to the earth, is called the angle of *Commutation.*

(53.) The angle of *Position* is the angle at an heavenly body formed by two great circles, one passing through the pole of the equator, and the other through the pole of the ecliptic.

(54.) *Apparent* noon is the time when the sun comes to the meridian.

(55.) *True* or *mean* noon is 12 o'clock, by a clock adjusted to go 24 hours in a mean solar day.

(56.) The *Equation of time* at noon, is the interval between *true* and *apparent* noon.

(57.) A star is said to rise or set *Cosmically,* when it rises or sets at sun-rising; and when it rises or sets at sun-setting, it is said to rise or set *Achronically.*

(58.) A star rises *Heliacally,* when, after having been so near to the sun as not to be visible, it emerges out of the sun's rays, and just appears in the morning; and it sets *Heliacally,* when the sun approaches so near to it, that it is about to immerge into the sun's rays and to become invisible in the evening.

(59.) The *curtate* distance of a planet from the sun or earth, is the distance of the sun or earth from that point of the ecliptic where a perpendicular to it passes through the planet.

(60.) *Aphelion* is that point in the orbit of a planet which is furthest from the sun.

(61.) *Perihelion* is that point in the orbit of a planet which is nearest the sun.

(62.) *Apogee* is that point of the earth's orbit which is furthest from the sun, or that point of the moon's orbit which is furthest from the earth.

(63.) *Perigee* is that point of the earth's orbit which is nearest the sun, or that point of the moon's orbit which is nearest the earth.

The terms aphelion and perihelion are also applied to the earth's orbit.

(64.) *Apsis* of an orbit is either it's aphelion or perihelion, apogee or perigee; and the line which joins the apsides is called *the line of the apsides.*

(65.) *Anomaly* (*true*) of a planet, is it's angular distance at any time from it's aphelion, or apogee—(*mean*) is the angular distance from the same point at the same time, if it had moved uniformly with it's mean angular velocity.

(66.) *Equation of the center* is the difference between the *true* and *mean* anomaly; this is sometimes called the *prosthapheresis.*

(67.) *Nonagesimal degree* of the ecliptic, is that point which is highest above the horizon.

(68.) The *mean* place of a body, is the place where a body (not moving with an uniformly angular velocity about the central body) *would* have been, if it had moved with it's mean angular velocity. The *true* place of a body, is the place where the body actually is at any time.

(69.) *Equations* are corrections which are applied to the *mean* place of a body, in order to get it's *true* place.

(70.) A *Digit* is a twelfth part of the diameter of the sun or moon.

(71.) Those bodies which revolve about the sun in orbits nearly circular, are called *Planets*, or *primary* planets for the sake of distinction; and those bodies which revolve about the *primary* planets are called *secondary* planets, or *Satellites*.

(72.) Those bodies which revolve about the sun in very elliptic orbits are called *Comets*. The sun, planets, and comets, comprehend all the bodies in what is called the *Solar System*.

(73.) All the other heavenly bodies are called *Fixed Stars*, or simply *Stars*.

(74.) *Constellation* is a collection of stars contained within some assumed figure, as a ram, a dragon, an hercules, *&c.* the whole heaven is thus divided into constellations. A division of this kind is necessary, in order to direct a person to any part of the heavens which we want to point out.

Characters used for the Sun, Moon and Planets.

☉	The Sun.	⚴	Pallas.
☽	The Moon.	⚵	Juno.
☿	Mercury.		Vesta.
♀	Venus.	♃	Jupiter.
⊖	The Earth.	♄	Saturn.
♂	Mars.	♅	Georgian.
⚳	Ceres.		

Characters used for the Days of the Week.

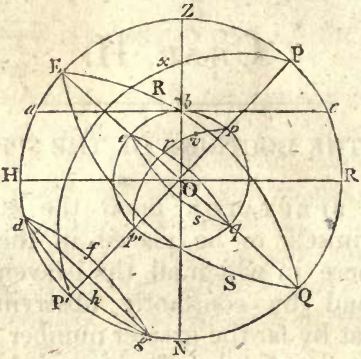
☉	Sunday.	♃	Thursday.
☽	Monday.	♀	Friday.
♂	Tuesday.	♄	Saturday.
☿	Wednesday.		

CHAP. II.

ON THE DOCTRINE OF THE SPHERE.

(75.) A SPECTATOR upon the earth's surface conceives himself to be placed in the center of a concave sphere in which all the heavenly bodies are situated; and by constantly observing them, he perceives that by far the greater number never change their relative situations, each rising and setting at the same interval of time, and at the same points of the horizon, and are therefore called *fixed* stars; but that a few others, called *planets*, together with the *sun* and *moon*, are constantly changing their situations, each continually rising and setting at different points of the horizon, and at different intervals of time. Now the determination of the times of the rising and setting of all the heavenly bodies; the finding of their position at any given time in respect to the horizon or meridian, or the time from their position; the causes of the different lengths of days and nights, and the changes of seasons; the principles of dialling, and the like, constitute the *doctrine of the sphere.* And as the apparent diurnal motion of all the bodies has no reference to any particular system, or disposition of the planets, but may be solved, either by supposing them actually to perform those motions every day, or by supposing the earth to revolve about an axis, we will suppose this latter to be the case, the truth of which will afterwards appear.

(76.) Let $pep'q$ represent the earth, O it's center, b the place of a spectator, $HZRN$ the sphere of the fixed stars; and although the fixed stars do not lie in the concave surface of a sphere of which the center of the earth is the center, yet, on account of the immense distance even of the nearest of them, their



relative situations from the motion of the earth, and consequently the place of a body in our system referred to them, will not be affected by this supposition. The plane *abc* touching the earth in the place of the spectator, is called (21) the *sensible* horizon, as it divides the visible from the invisible part of the heavens; and a plane *HOR* parallel to *abc*, passing through the center of the earth, is called the *rational* horizon; but in respect to the sphere of the fixed stars, these may be considered as coinciding, the angle which the arc *Ha* subtends at the earth becoming then insensible, from the immense distance of the fixed stars. Now if we suppose the earth to revolve daily about an axis, all the heavenly bodies must successively rise and set in that time, and appear to describe circles whose planes are perpendicular to the earth's axis, and therefore parallel to each other, because each body continues at the same distance from the equator, during the revolution of the earth about it's axis. Thus, all the stars will appear to revolve daily about the earth's axis, as if they were placed in the concave surface of a sphere having the earth in the center. Let therefore *pp'* be that diameter of the earth about which it must revolve in order to give the apparent diurnal motion to the heavenly bodies, then *p*, *p′*, are called it's poles; and if *pp′* be produced both ways to *P*, *P′*, in the heavens, these points are called (18) the poles of the heavens, and the star nearest to each of these is called the pole star. Now, although the earth, from it's motion in it's orbit, continually changes it's place, yet as the axis always continues parallel to itself, the points *P*, *P′*, will not, from the immense distance of the fixed stars, be sensibly altered; we may therefore suppose these to be fixed points*. Produce *Ob* both ways to *Z* and *N*, and *Z* is the zenith, and *N* the nadir (23). Draw the great circle *PZHNR*, and it will be the celestial meridian (25), the plane of which coincides with the terrestrial meridian *pbp′* passing through the place *b* of the spectator. Let *erqs* represent a great circle of the earth perpendicular to it's axis *pp′*, and it will be the equator (15); and if the plane of this circle be extended to the heavens, it marks out a great circle *ERQS* called the celestial equator (18). Hence, for the same reason that we may consider the points *P*, *P′*, as fixed, we may consider the circle *ERQS* as fixed. Now as the latitude of any place *b* on the earth's surface is measured by the degrees of the arc *be* (16), it may be measured by the degrees of the arc *ZE*; hence, as the equator, zenith, and poles in the heaven, correspond to the equator, place of the spectator, and poles of the earth, we may leave out the consideration of the earth in our further enquiries upon this subject, and only consider the equator, zenith, and poles in the heavens, and *HR* the rational horizon to the spectator.

(77.) Let the annexed figure represent the position of the heavens to Z the zenith of a spectator in *north* latitude, EQ the equator, P, P', it's poles, HOR

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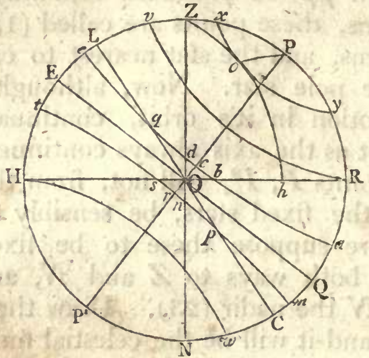
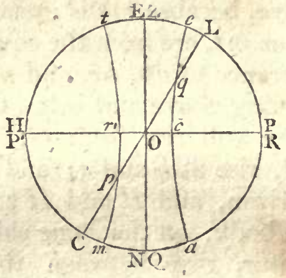
* This is not accurately true, the earth's axis varying a little from it's parallelism from the action of the moon. This is called the *Nutation* of the earth's axis, and was discovered by Dr. BRADLEY.



the rational horizon, $PZHP'R$ the meridian of the spectator, and draw the great circle ZON perpendicular to $ZPRH$, and it is the prime vertical (28); R will be the north point of the horizon, and H the south (26), and O will be the east or west point, (28) according as this figure represents the eastern or western hemisphere. Draw also a great circle POP' perpendicular to the meridian. We must therefore conceive this figure to represent half a globe, and all the lines upon it to represent circles; and if we conceive the eye to be vertical to the middle point O of the figure, all the circles which pass through that point will appear right lines; therefore the right lines ZON, POP, EOQ, HOR, must be considered as semicircles. Now as each circle HR, EQ, ZN, PP' is perpendicular to the meridian, it's pole must be in each (8, 9), therefore their common intersection O is the pole of the meridian. Draw also the small circles wH, mt, ae, Rv, yx, parallel to the equator; and as the great circle POP' bisects EQ in O, it must also bisect the small circles mt, ae, in r and c; for as $EO=90^\circ$, tr and ec are each $=90^\circ$ (13); and as $QO=90^\circ$, mr and ac are each $=90^\circ$; hence, $ac=ce$, and $mr=rt$.

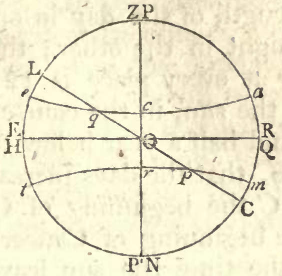
(78.) As all the heavenly bodies, in their diurnal motion, describe either the equator, or small circles parallel to the equator, according as the body is in or out of the equator; if we conceive this figure to represent the eastern hemisphere, *QE*, *ae*, *mt*, may represent their apparent paths from the meridian under the horizon to the meridian above, and the points *b*, *O*, *s*, are the points of the horizon where they rise. And as *ae*, *QE*, *mt*, are bisected in *c*, *O*, *r*, *eb* must be greater than *ba*, *QO* equal to *OE*, and *ts* less than *sm*. Hence, a body on the *same* side of the equator with the spectator will be longer above the horizon than below, because *eb* is greater than *ba*; a body *in* the equator will be as long above as below, because $QO = OE$; and a body on the *contrary* side will be longer below than above, because *ms* is greater than *st*. And the further *ae*, or *mt*, are from the equator, the greater will be the difference of *ab*, *be*, and *ms*, *st*, or of the times of continuing above and below the horizon; and the further they will rise from *O*. The bodies describing *ae*, *mt*, rise at *b* and *s*; and as *O* is the east point of the horizon, and *R* and *H* are the north and south points, a body, on the *same* side of the equator with the spectator, rises between the east and the north, and a body on the *contrary* side rises between the east and the south, the spectator being supposed to be in *north* latitude; and a body *in* the equator rises in the east at *O*. When the bodies come to *d* or *n*, they are in the prime vertical, or in the east; hence, a body on the *same* side of the equator with the spectator comes to the east *after* it is risen, and a body on the *contrary* side, *before* it rises. The body which describes the circle *Rv*, or any circle nearer to *P′*, never sets; and such circles are called circles of *perpetual apparition*; and the stars which describe them are called *circumpolar* stars. The body which describes the circle *wH*, just becomes visible at *H*, and then it instantly descends below the horizon; but the bodies which describe the circles nearer to *P′* are never visible. Such is the apparent diurnal motion of the heavenly bodies, when the spectator is situated any where between the equator and poles; and this is called an *oblique* sphere, because all the bodies rise and set obliquely to the horizon. As this figure may also represent the western hemisphere, the same circles *ea*, *tm* will represent the motions of the heavenly bodies as they descend from the meridian above the horizon to the meridian under. Hence, a body is at the greatest altitude above the horizon, when on the meridian, and at equal altitudes when equidistant on each side, from it, if the body have not changed it's declination.

(79.) If the spectator be at the *equator*, then *E* coincides with *Z*, and consequently *EQ* with *ZN*, and



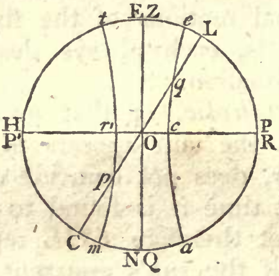
therefore *PP′* with *HR*. Hence, as the equator *EQ* is perpendicular to the horizon, the circles *ace*, *mrt*, parallel to *EQ* must also be perpendicular to it; and as these circles are always bisected by *PP′*, they must now be bisected by *HR*. Hence, all the heavenly bodies are as long above the horizon as below, and rise and set at right angles to it, on which account this is called a *right* sphere.

(80.) If the spectator be at the *pole*, then *P* coincides with *Z*, and consequently *PP′* with *ZN*, and therefore *EQ* with *HR*. Hence, the circles *mt*, *ae*, parallel to the equator, are also parallel to the horizon; therefore as a body in it's diurnal motion describes a circle parallel to the horizon, those fixed bodies in the heavens, which are above the horizon, must always continue above, and those which are below must always continue below. Hence, none of the bodies,



by their *diurnal* motion, can either rise or set. This is called a *parallel* sphere, because the diurnal motion of all the bodies is parallel to the horizon. These apparent diurnal motions of the fixed stars remain constant, that is, each always describes the same parallel of declination.

(81.) The *ecliptic*, or that great circle in the heavens which the sun appears to describe in the course of a year, does not coincide with the equator, for during that time it is found to be only twice in the equator; let therefore *COL* represent half the ecliptic, or half the sun's apparent annual motion; *C* the first point of Capricorn, and *L* the first point of Cancer; and this being a great circle, must cut the equator into two equal parts (7). Hence, as the apparent motion of the sun is nearly uniform, the sun is nearly as long on one side of the equator as on the other. (See Fig. in p. 16.) When therefore the sun is at *q*, on the *same* side of the equator with the spectator, describing the parallel of declination *ae* by it's apparent diurnal motion, the days are longer than the nights, and it rises at *b* to the north of the east point; but when it is on the *contrary* side, at *p*, describing *mt*, the days are shorter than the nights, and it rises at *s* to the south of the east point, the spectator being on the *north* side of the equator; but when the sun is *in* the equator, at *O*, describing *QE*, the days and nights are equal, and it rises in the east, at *O**. If *ae, mt* be equidistant from *EQ*, then will $be = ms$, and $ab = st$; hence, when the sun is in these opposite parallels, the length of the day in one is equal to the length of the night in the other; therefore the *mean* length of a day at every place is 12 hours. Hence, at every place, the sun, in the course of a year, is half a year above, and half a year below the horizon†. It is manifest, also, that the days increase from the time the sun leaves *C* the beginning of Capricorn, till he comes to *L* the beginning of Cancer; and that they decrease from the time the sun leaves the beginning of Cancer till he comes to the beginning of Capricorn. When the spectator is at the *Equator*, the sun at *p* or *q* describing the circles *mt, ae*, by it's apparent *diurnal* motion, and these being bisected by the horizon, the sun will be always as long above as below the horizon,



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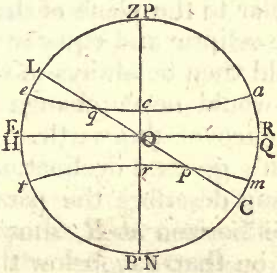
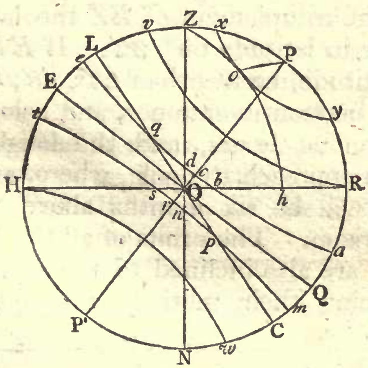
* The different degrees of heat in summer and winter, do not altogether arise from the different times which the sun is above the horizon, but partly from the different altitudes of the sun above the horizon; the higher the sun is above the horizon, the greater is the number of rays which fall on any given space, and the greater also is the force of the rays. From all these circumstances arise the different degrees of heat in summer and winter. The increase of heat also as you approach the equator, arises from the two latter circumstances.

† This is not accurately true, because the sun's motion in the ecliptic is not quite uniform, on which account it is not exactly as long on one side of the equator as on the other. If the major axis of the earth's orbit coincided with the line joining the equinoctial points, the times would be equal.

and consequently the days and nights will be always 12 hours long. There will however be some variety of seasons, as the sun will recede 23°. 28′ on each side from the spectator. In this situation of the spectator, the sun will be vertical to him at noon when it is in the equator. And when the spectator is any where between the tropics, the sun will be vertical to him at noon, when it's declination is equal to the latitude of the place, and of the same kind, that is, when they are both north, or both south. When the spectator is at the *Pole,* the sun at *p* or *q* is carried, by it's apparent *diurnal* motion, in the circles *mpt, aqe,* parallel to the horizon; hence, it never sets when it is in that part *OL* of the ecliptic which is above the horizon, nor rises when in that part *OC* which is below; consequently there is half a year day, and half a year night. As the sun illuminates one half of the earth, or 90° all round about that place to which he is ver-





tical*, when he is *in* the equator, he will illuminate as far as each pole; when he is on the north side of the equator, the north pole will be within the illuminated part, and the south pole will be in the dark part; and when the sun is on the south side of the equator, the south pole will be within the illuminated part, and the north pole in the dark part. And when the sun is at the tropic, he illuminates 23°. 28′ beyond one pole, and the other pole is 23°. 28′ within the dark part. Hence, the variety of seasons arises from the axis of the earth, which coincides with PP', not being perpendicular to the plane of the ecliptic LOC, for if it were, the ecliptic and equator would coincide, and the sun would then be always in the equator, and consequently it would never change it's position in respect to the surface of the earth. If $QR=EH=$ 23°. 28′, the sun's greatest declination, then on the longest day the sun describes the parallel Rv, which just touching the horizon at R, shows that the sun does not descend on that day below the horizon, and therefore that day is 24 hours long. But when the sun comes to it's greatest declination on the other side of EQ, it describes wH, and consequently does not ascend above the horizon for 24 hours, and therefore that night is 24 hours long. This therefore happens when EH, the complement of EZ the latitude (16), is 23°. 28′, or in latitude 66°. 32′. If EH, the complement of latitude, be less than 23°. 28′, the sun will be above the horizon in summer, and below in winter, for more than 24 hours, and the longer above or below, as you approach the pole, where, as was before observed, it will be six months above, and as long below the horizon. The orbits of all the planets, and of the moon, are also inclined to the equator, as appears by tracing their motions amongst the fixed stars; therefore, in the time in which each makes one revolution in it's orbit, the same appearances will take place, as in the sun. All these different appearances in the motion of the moon, must therefore happen in every month. It is also evident, that these variations of rising and setting must be greater or less, as the orbits are more or less inclined to the equator, as appears by Art. 78. Hence, they must be greater in the moon than in the sun*. The apparent annual motion of the sun, and the real motion of the moon and planets, is from west to east, and therefore contrary to their apparent diurnal motion.

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* This is not accurately true, because, as the sun is greater than the earth, he will illuminate beyond 90°, by a quantity which is nearly equal, in minutes of a degree, to his apparent semidiameter.

(82.) Hitherto we have considered the motion of the heavenly bodies in the eastern hemisphere; but if the figure represent the western hemisphere, all the reasoning will equally apply. The bodies will be just as long in descending from the meridian to the horizon, as in ascending from the horizon to the meridian; the paths described will be similar; and they will set in the same situation in respect to the west point of the horizon, as they rise in respect to the east; that is, if a body rise to the north or south of the east, it will set at the same distance from the west towards the north or south.

(83.) Having thus explained all the apparent diurnal motions of the heavenly bodies, with the cause of the variety of seasons, we shall proceed in the next place to show the method of determining the positions of the different circles, and the situation of the bodies in respect to the horizon, meridian, or any other circles, at any given time; and having given their situation, to find the time; for the understanding of which a knowledge of plane and spherical trigonometry is all that is requisite.

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* On account of the continual change of declination of the sun, moon, and planets, their apparent diurnal motions will not be accurately parallel to the equator; in those cases therefore, where the declination alters sensibly in the course of a day, and where great accuracy is required, we must, in our computations, take into consideration, the change of declination.

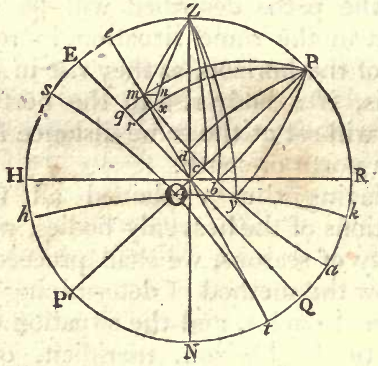
(84.) *The altitude* PR *of the pole above the horizon, is equal to the latitude of the place.*

For the arc ZE is (16) the measure of the latitude; but $PE=ZR$, each being $=90°$; take away ZP which is common to both, and $EZ=PR$*.

(85.) *To find the latitude of a place, observe the greatest and least altitude of a circumpolar star, and apply the correction for refraction, in order to get the true altitudes, and half the sum will be the altitude of the pole.*

For if yx be the true circle described by the star, then, as $Px=Py$, $PR=\frac{1}{2}\times\overline{Ry+Rx}$. See the last figure.

The latitude may also be thus found.



Let *eOt* be the ecliptic; then when the sun comes to *e*, it's declination is the greatest, and *eH* is the greatest meridian altitude; when the sun comes to the ecliptic at *t*, let *ts* be the parallel described on that day, and then sH is the least meridian altitude; and as $Ee = Es$, $\frac{1}{2} \times \overline{He + Hs} = HE$ the complement of the latitude.

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* From hence arises the method of measuring the circumference of the earth; for if a man travel upon a meridian till the height of the pole has altered one degree, he must then have travelled one degree; hence, by measuring that distance and multiplying it by 360, we get the circumference of the earth. This was undertaken by our countryman Mr. *Norwood*, who measured the distance between London and York, and observed the different altitudes of the pole at those places. Afterwards, the French mathematicians measured

(86.) *Half the difference of the sun's greatest and least meridian altitudes, is equal to the inclination of the ecliptic to the equator.*

For half $\overline{He - Hs}$, or half se, is equal to Ee which (12) measures the angle EOe, the inclination of the ecliptic to the equator.

(87.) *The angle which the equator makes with the horizon, or the altitude of that point of the equator which is on the meridian, is equal to the complement of the latitude.*

For ZH is 90°, and therefore EH is the complement of EZ; and as $OE = OH = 90°$, EH measures (12) the angle EOH*.

(88.) Let $abcdxe$ be a parallel of declination described by an heavenly body in the eastern hemisphere, and draw the circles of declination Pb, Pc, Pd, Px, and the circles of altitude Zb, Zc, Zd, Zx. Now, as has been already explained, when the body comes to b, it rises; at c it is at the middle point between a and e; and at d it is due east; and let x be it's place at any other time. Let us suppose this body to be the sun, and not to change it's declination in it's passage from a to e, and let us suppose a clock to be adjusted to go 24 hours in one apparent diurnal revolution of the sun, or from the time it leaves any meridian till it returns to it again, then the sun will always approach the meridian, or any other circle of declination, at the

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measured a degree. *Cassini* measured one in France. After that, *Clairaut*, *Maupertuis*, and several other mathematicians went to Lapland, and measured a degree, the length of which appears to be 69,2 English miles in the latitude of 45°; for the earth being a spheroid, the degrees in different latitudes are different.

* See my *Treatise on Plane and Spherical Trigonometry*, Art. 173. This is the Trigonometry referred to in the future part of this Work.

rate of 15° in an hour; also, the angle which the sun describes about the pole will vary at the same rate, because (13) an arc xe, which the sun at x has to describe before it comes to the meridian, measures the angle xPe, called the *hour* angle. If therefore we suppose the clock to show 12 when the sun is on the meridian at a or e, it will be 6 o'clock when he is at c. And as the sun describes angles about the pole P at the rate of 15° in an hour, the angle between any circle, Px, of declination passing through the sun at x, and the meridian PE, converted into time at the rate of 15° for an hour, will give the time from *apparent* noon, or when the sun comes to the meridian.

(89.) *Given the sun's declination, and latitude of the place, to find the time of his rising, and azimuth at that time.*

The sun rises at b; and in the triangle bZP, $bZ = 90°$, $bP =$ co-dec. $PZ =$ co-lat. Now when one side of a triangle $= 90°$, it may be solved by the circular parts, taking the angles adjacent to the side $= 90°$, and the complements of the other three parts, for the circular parts. Hence, (Trig. Art. 215.) rad. × cos. $ZPb =$ cot. bP × cot. ZP, or, rad. × cos. hour angle = tan. dec. × tan. lat. therefore (Trig. Art. 213).

Log.tan.dec.+log.tan.lat.−10,=*log.cos.hour ang.from app.noon;*

which converted into time, at the rate of 15° for an hour, (see Table I. at the end), and subtracted from 12 o'clock, gives the apparent time of rising. Also, (Trig. Art. 215) rad. × cos. $bP =$ sin. ZP × cos. PZb, or, rad. × sin. dec. = cos. lat. × cos. azi. therefore

10,+*log. sin. dec.*−*log. cos. lat.*=*log. cos. azi. from North.*

Ex. Given the latitude of Cambridge 52°. 12′. 35″, to find the time of the sun's rising on the longest day, and azimuth at that time, assuming the greatest declination of the sun 23°. 28′.

Dec. 23°. 28′. 0″. - tan. 9,6376106
Lat. 52°. 12′. 35. - tan. 10,1104699

Hour ∠ 124°. 2′. 27″[*]. - cos. 9,7480805

Convert this into time (Tab. I.) and it gives 8*h*. 19′. 6″, which subtracted from 12, gives 3*h*. 40′. 54″, the time when the sun's center is upon the rational horizon on the longest day, Also,

Dec. 23°. 28′. 0″. - 10, + sin. 19,6001181
Lat. 52. 12. 35. - - cos. 9,7872996

Azi. 49. 28. 9. - - cos. 9,8128185

Hence, on the longest day, the sun rises 40°. 31′. 51″ from the east towards the north.

(90.) *To find the sun's altitude at six o'clock.*

The sun is at c at 6 o'clock, and the angle ZPc is a right one; hence, (Trig. Art. 212) rad. × cos. $Zc =$ cos. ZP × cos. Pc, or, rad. × sin. alt. = sin. lat. × sin. dec. therefore

Log. sin. lat. + log. sin. dec. − 10, = *log. sin. alt.*

Ex. Taking the data of the last example, we have,

Lat. 52°. 12′. 35″ - - sin. 9,8977695
Dec. 23. 28. 0 - - sin. 9,6001181

Alt. 18. 20. 32. - - sin. 9,4978876

(91.) *To find the time when the sun comes to* d *the prime vertical, and it's altitude at that time.*

In this case, the angle $dZP = 90°$; hence, (Trig. Art. 112) rad. × cos. $dP =$ cos. ZP × cos. Zd, or, rad. × sin. dec. = sin. lat. × sin. alt. therefore

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[*] This log. 9,7480805 is found in the tables to be the log. cosine of 55°. 57′. 13″, but as the angle is manifestly greater than 90°, we must take it's supplement. In the solution of spherical triangles, ambiguous cases will frequently arise; for the determination of which, where the case is not evident, the reader is referred to my *Treatise on Trigonometry*.

10,+ *log. sin. dec.* − *log. sin. lat* = *log. sin. alt.*

Also, (Trig. Art. 212) rad. × cos. ZPd = cot. Pd × tan. PZ, or, rad. × cos. hour angle = tan. dec. × cot. lat. therefore

log. tan dec. + *log. cot. lat.* − 10, = *log. cos. hour angle;* which, converted into time (Tab. I.), gives the time from *apparent* noon.

Ex. Taking the data of the last example, we have,

Dec. 23°. 28′. 0″	-	10+,sin. 19,6001181
Lat. 52. 12.35	- -	sin. 9,8977695
Alt. 30. 15.31	- -	sin. 9,7023486
Dec. 23°. 28′. 0″	- -	tan. 9,6376106
Lat. 52. 12.35	- -	cot. 9,8895301
Hour∠70. 19. 44.	- -	cos. 9,5271407

This angle 70°. 19′. 44″, converted into time, gives 4*h*. 41′. 19″ the time from *apparent* noon.

(22.) *Given the latitude of the place, the sun's declination, and altitude, to find the hour, and his azimuth.*

Let x be the sun's place; then, (Trig. Art. 239) sin. $Px \times$ sin. PZ : rad.^{2} :: sin. $\frac{1}{2} \times \overline{Px+PZ+Zx} \times$ sin. $\frac{1}{2} \times \overline{Px+PZ-Zx}$: $\overline{\text{cos. } \frac{1}{2}\, ZPx}^2$; hence, ZPx is known, which converted into time (Tab. I.), gives the time from *apparent* noon. Also, (Trig. Art. 239) sin. $Zx \times$ sin. ZP : rad.^{2} :: sin. $\frac{1}{2} \times \overline{Zx+ZP+Px} \times$ sin. $\frac{1}{2} \times \overline{Zx+ZP-Px}$: $\overline{\text{cos. } \frac{1}{2}\, xZP}^2$; hence, the azimuth xZP from the north is known.

Ex. Given the lat. 34°. 55′ N, sun's declination 22°. 22′. 57″ N, and true altitude 36°. 59′. 39″, to find the apparent time.

Here, ZP = 55°. 5′, Zx = 53°. 0′. 21″, Px = 67°. 37′. 3″; hence (Trig. Art. 239)

$Px = 67°. 37'. 3''$ - ar. co. sin. 0,034019
$ZP = 55. 5. 0$ - ar. co. sin. 0,086193
$Zx = 53. 0. 21$

Sum 175. 42. 24

$\frac{1}{2}$ Sum 87. 51. 12 - - - sin. 9,999694
$Zx = 53. 0. 21$

Diff. 34. 50. 51 - - - sin. 9,756932

2)19,876838

9,938419

the cosine of $29°. 47'. 44''$, half the angle ZPx, $\therefore ZPx = 59°. 35'. 28''$, which reduced into time gives $3h. 58'. 22''$, the time from *apparent* noon. By the very same process, the angle xZP is found.

(93.) *Given the error in altitude, to find the error in time.*

Let mn be parallel to the horizon, and nx represent the error in altitude; then, as the calculation of the time is made upon supposition that there is no error in the declination, we must suppose the body to be at m instead of x, and consequently the angle mPx, or the arc qr, measures the error in time.

$$\text{Now } nx : xm :: \sin.\ nmx : \text{rad.}\ \text{(Trig. Art. 125)}$$
$$\underline{xm : qr :: \cos.\ rx\ : \text{rad.}}\ \text{(Art. 13).}$$

hence, $nx : qr :: \sin.\ nmx \times \cos.\ rx : \text{rad.}^2 \therefore qr = nx \times \dfrac{\text{rad.}^2}{\sin.\ nmx \times \cos\ rx}$; but $ZxP = nmx$, nxm being the complement of both; also, (Trig. Art. 221) $\sin.\ ZxP$, or nmx, : $\sin.\ ZP :: \sin.\ xZP : \sin.\ xP$, or $\cos.\ rx$, $\therefore \sin.\ nmx \times \cos.\ rx = \sin.\ ZP \times \sin.\ xZP$; hence, $qr =$

$$nx \times \frac{\text{rad.}^2}{\sin.\ ZP \times \sin.\ xZP} = nx \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}.$$

Hence, the error is least on the prime vertical. All altitudes therefore, for the purpose of deducing the time, ought to be taken on, or as near to, the prime vertical as possible.

Ex. In lat. 50°. 12′, if the error in alt. at an azim. 44°. 22′ be 1′, then $qr = 1' \times \frac{1^2}{,612 \times ,690} = 2',334$ of a degree $= 9'',336$ in time.

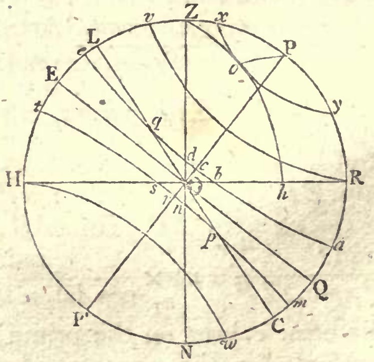
Hence, the perpendicular ascent of a body is quickest when it is on the prime vertical; for nx varies as sin. azim. when qr and the lat. are given.

(94.) *Given the lat. of the place, and the sun's declination, to find the time when twilight begins.*

Twilight is here supposed to begin when the sun is 18° below the horizon; draw therefore the circle *hyk* parallel to the horizon, and 18° below it, and twilight will begin when the sun comes to y, and $Zy = 108°$; hence, (Trig. Art. 239) $\sin. Py \times \sin. PZ : \text{rad.}^2 :: \sin. \frac{1}{2} \times \overline{PZ + Py + 108°} \times \sin. \frac{1}{2} \times \overline{PZ + Py - 108°} : \cos. \frac{1}{2}\, yPZ^2$; therefore yPZ is known, which converted into time (Tab. I.), gives the time from *apparent* noon. The operation is the same as that in Art. 92.

(95.) *To find the time when the apparent diurnal motion of a fixed star, is perpendicular to the horizon.*

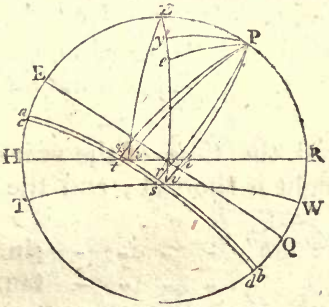
Let yx be the parallel described by the star; draw the vertical circle Zh, touching it at o, and when the star comes to o, it's motion is perpendicular to the horizon; and as ZoP is a right angle, we have (Trig.



Art. 212) rad. × cos. ZPo = tan. Po × cot. PZ, or, rad. × cos. hour angle = cot. dec. × tan. lat. therefore, *log. cot. dec. + log. tan. lat.* – 10, = *log. cos. hour angle*; which converted into time (Tab. I.), gives the time from the star's being on the meridian. Hence, the time of the star's coming to the meridian being known, the time required will be known.

(96.) *To find the time of the shortest twilight.*

Let ab be the parallel of the sun's declination at the time required, draw cd indefinitely near and parallel to it, and TW a parallel to the horizon 18° below it; then vPw, sPt measure the duration of twilight on each parallel of declination, and when the twilight is shortest, the increment of the duration is $=0$, and these must be equal; hence, $vPr = wPz$, therefore $vr = wz$; and as $rs = tz$, and r and z are right angles, $rvs = zwt$; but $Pvr = 90° = Zvs$, take Zvr from both, and $PvZ = rvs$; for the same reason $PwZ = zwt$; hence, $PvZ = PwZ$. Take $ve = wZ = 90°$, and join Pe; and as $Pv = Pw$, $ve = wZ$, and $Pve = PwZ$, we



have $Pe = PZ$; and if Py be perpendicular to eZ, then will $Zy = ye$. Now, (Trig. Art. 224) cos. Pv : cos. Pe, or PZ, :: cos. vy : cos. ey, that is, sin. dec. : sin. lat. :: sin. ey : cos. ey :: tan. $ey = 9°$: rad. or, *rad. : sin. lat. :: tan.* 9° *: sin. of the sun's declination* at the time of the shortest twilight; and the logarithmic operation is,

log. sin. lat. + log. tan. 9° – 10, = *log. sin. dec.*

Because PZ is never greater than 90°, and $Zy = 9°$, therefore Py is never greater than 90°, and it's cosine is positive; also, vy is always greater than 90°, therefore it's cosine is negative; hence, (Trig. Art. 212. rad. being unity) cos. Pv ($=$ cos. $Py \times$ cos. vy) is negative; consequently Pv is greater than 90°; therefore the sun's declination is *south.*

If, instead of taking $RW = 18°$, we take it $=$ the sun's diameter ($2s$), we shall get the time of the year when the body of the sun is the least time in ascending above the horizon; hence,

$$\textit{log. sin. lat.} + \textit{log. tan. } s - 10, = \textit{log. sin. dec.}$$

Thus we get the declination when the sun is the least time in rising; and as the declination must be always very small, this event must happen when the sun is very near the equinox.

(97.) *To find the duration of the shortest twilight.*

As $wPz = vPe$, therefore $ZPe = vPw$, which measures the shortest time. Now, (Trig. Art. 212) rad. $\times$ sin. $Zy =$ sin. $PZ \times$ sin. ZPy, or, rad. $\times$ sin. $9° =$ cos. lat. $\times$ sin. ZPy, therefore,

$$10, + \textit{log. sin. } 9° - \textit{log. cos. lat.} = \textit{log. sin. } ZPy,$$

which doubled gives ZPe, or vPw, which, converted into time (Tab. I.), gives the duration of the shortest twilight.

Ex. To find the time of the year at *Cambridge*, when the twilight is shortest; and the length of that twilight.

Lat. 52°. 12′. 35″	sin.	9,8977695
9°	tan.	9,1997125
Dec. 7°. 11′. 25″	sin.	9,0974820

This declination of the sun gives the time about March 2, and October 11.

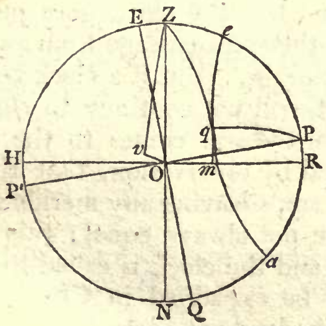
9°. 0′. 0″	10, + sin.	19,1943324
Lat. 52. 12. 35	cos.	9,7872996
ZPy 14. 47. 27	sin.	9,4070328

The double of this gives 29°. 34′. 54″, which, converted into time, gives 1*h*. 58′. 20″ for the duration of the shortest twilight, it being supposed to end when the sun is 18° below the horizon.

(98.) *To find the sun's declination, when it is just twilight all night.*

Here the sun at a (Fig. p. 25) must be 18° below the horizon; therefore 18°+dec. $Qa=RQ=EH=$ comp. of lat. of place; hence, the sun's dec.=comp. lat.−18°; look therefore into the *Nautical Almanac*, and see on what days the sun has this declination, and you have the time required. The sun's greatest declination being 23°. 28′, it follows, that if the complement of latitude be greater than 41°. 28′, or if the latitude be less than 48°. 32′, there can never be twilight all night. If the sun be on the other side of the equator, then it's dec.=18°.−comp. lat.

(99.) If the sun's declination Ee be greater than EZ, then the sun comes to the meridian at e to the *north* of the zenith Z of the spectator; and if we draw the secondary Zqm touching the parallel ae of declination described by the sun, then Rm is the greatest azimuth from the north which the sun has that day, the



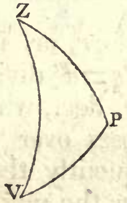
azimuth increasing till the sun comes to q, and then decreasing; for a circle from Z to any other point of ea will cut RO nearer to R, and it will also cut ea in two points which have the same azimuth, they being in the same vertical circle; in this case, therefore, the sun has the same azimuth twice in the morning. If, therefore, we draw the straight line Zv perpendicular to the horizon, the shadow of this line, being always opposite to the sun, will, in the morning, first recede from the south point H, and then approach it, and therefore will go backwards upon the horizon. But if we consider PP' as a straight line, or the earth's axis produced, the shadow of that line will not go backwards upon the horizon, because the sun always revolves about that line, whereas it does not revolve about the perpendicular Zv, it never getting to the south of it. Hence it appears, that the shadow of the sun upon a dial can never go backwards, because the gnomon of a dial is parallel to PP', and therefore the sun must always revolve about the gnomon.

The time when the azimuth is greatest is found from the right angled triangle PqZ; for (Trig. Art. 212) rad. × cos. ZPq = tan. qP × cot. PZ, or, rad. × cos. hour angle = cot. dec. × tan. lat.; therefore,

log. cot. dec. + *log. tan. lat.* − 10, = *log. cos. hour angle* from *apparent* noon.

(100.) It has hitherto been supposed, that it is 12 o'clock when the sun comes to the meridian ZHN (Fig. p. 20) and that the clock goes just 24 hours in the interval of the sun's passage from any meridian till it returns to it again. But if a clock be thus adjusted for one day, it will not continue to show 12 o'clock every day when the sun comes to the meridian, because it is found by observation, that the intervals of time from the sun's leaving any meridian till it returns to it again, are not always equal; this difference between the sun and the clock is called the *Equation of Time*, as will be explained in Chap. IV. Hence, when the clock does not agree with the sun, and the sun is at x, any arc xe is not the measure of the time from 12 o'clock, but from the time when the sun comes to the meridian, or from *apparent* noon, as it is called.

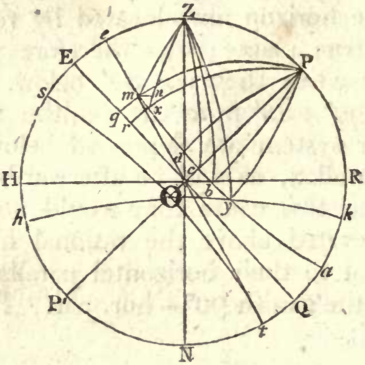
(101.) The method of finding the hour angle for the time at which a body rises, has been upon the supposition that the body is upon the rational horizon at the instant it appears, or 90° from the zenith; but all bodies in the horizon are elevated by refraction 33′ above their true places; this therefore would make them appear when they are 33′ below the rational horizon, or $90°+33'$ from the zenith; also, all the bodies in our system are depressed below their true places by parallax, as will be afterwards explained; therefore from this cause they would not appear till they were elevated above the rational horizon by a quantity equal to their horizontal parallax, or when distant from the zenith $90°-$hor. par. Hence, from



both causes together, a body becomes visible when it's distance ZV from the zenith $=90°+33'-$hor. parallax, V being the place of the body when it becomes visible, Z the zenith, and P the pole; hence, knowing ZV, also ZP the complement of latitude, and PV the complement of declination, we can find the hour angle ZPV. A fixed star has no parallax, therefore in this case $ZV=90°.\ 33'$.

To find the Time in which the Sun passes the Meridian, or the horizontal Wire of a Telescope.

(102.) Let mx be the diameter d'' of the sun, estimated in seconds of a great circle; then, as the seconds in mx, considered as a small circle, must be increased in proportion as the radius is diminished, because (Trig. Art. 75) when the arc is given, the angle is inversely as the radius, we have, sin. Px, or cos. dec. rx, : rad. :: seconds d'' in mx of a great circle : the seconds in mx of the small circle ea, which (13) is



equal to the seconds in qr, or, in the angle rPq, and therefore the angle $rPq = d''$ divided by cos. dec. (rad. being unity) $= d'' \times$ sec. dec., which measures the time in which the sun passes over a space equal to it's diameter, and consequently the time the diameter will be in passing over the meridian ; hence, 15″ in space (corresponding to 1″ in time) : $d'' \times$ sec. dec. in space :: 1″ in time : the time in seconds of passing the meridian $= \frac{d'' \times \text{sec. dec.}}{15''}$.

(103.) Hence, qr, the sun's diameter in right ascension, is equal to $d'' \times$ sec. dec. If, therefore, the sun's diameter $= 32' = 1920''$, and it's dec. $= 20°$, it's diameter in right ascension $= 1920'' \times 1{,}064 = 34'.\ 2{,}''88$. The same is true for the moon, if $d'' =$ it's diameter.

(104.) By Art. 93. $qr = nx \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}$

$= (\text{if } nx = d''\text{, the sun's diameter})\, d'' \times \frac{\text{rad}^2.}{\text{cos. lat.} \times \text{sin. azim.}}$;

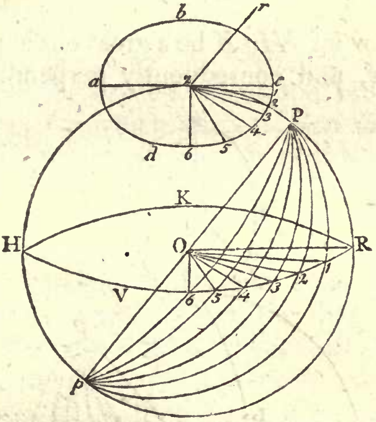
hence, as before, the time of describing qr, or the time in which the sun ascends perpendicularly through a space equal to it's diameter, or the time of passing an horizontal wire, is equal to $\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}$

The same expression must also give the time which the body of the sun is in ascending above the horizon.

If $d''=1980''$ the horizontal refraction, then d'' divided by $15''=132''$; hence, refraction accelerates the rising of the sun by $132''\times\frac{\text{rad.}^2}{\text{cos. lat.}\times\text{sin. azim.}}$.

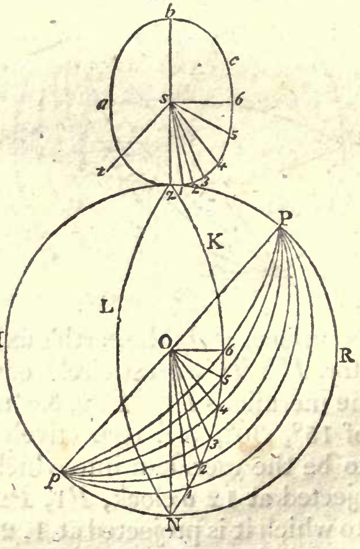
On the Principles of Dialling.

(105.) As the apparent motion of the sun about the axis of the earth, is at the rate of 15° in an hour, very nearly, let us suppose the axis of the earth to project it's shadow into the meridian opposite to that in which the sun is, and then this meridian will move at the rate of 15° in an hour. Hence, let $zPRpH$ represent a meridian on the earth's surface, POp the earth's axis, z the place of the spectator, $HKRV$ a great circle, of which z is the pole; draw the meridians $P1p$, $P2p$, &c. making angles with PRp of 15°, 30°, &c. respectively; then, supposing PR to be the meridian into which the shadow of OP is projected at 12 o'clock, $P1$, $P2$, &c. are the meridians into which it is projected at 1, 2, &c. o'clock, and the shadow will be projected on the plane $HKRV$ into the lines OR, $O1$, $O2$, &c. and the angles $RO1$, $RO2$, &c. will be the angles between the 12 o'clock line and the 1, 2, &c. o'clock lines. Now in the right



angled triangle *PR*1, we have (84) *PR* the latitude of the place, and the angle *RP*1 = 15°; hence, (Trig. Art. 210) *rad.* : *tan.* 15° :: *sin.* PR : *tan.* R1; in the same manner we may calculate the arcs *R*2, *R*3, &c. In this case, we make the earth's axis the gnomon, and the shadow is projected upon the plane *HKRV*. Take a plane *abcd* at *z*, parallel to *HKRV*, and it is the sensible horizon (21), and draw *zr* parallel to *POp*; then, on account of the great distance of the sun, we may conceive it to revolve about *zr* in the same manner as about *PO*, and consequently the shadow will be projected upon the plane *abcd*, in the same manner as the shadow of *PO* is projected upon the plane *HKRV*, and therefore the hour angles are calculated by the same proportion. This is an *horizontal* dial.

(106.) Now let *NLzK* be a great circle perpendicular to *PRpH*, and consequently perpendicular to the



horizon at *z*, and the side next to *H* is full south. Then, for the same reason as before, if the angles *Np*1, *Np*2, &c. be 15°, 30°, &c. the shadow of *pO* will be projected into the lines $O1$, $O2$, &c. at 1, 2, &c. o'clock, and the angles $NO1$, $NO2$, &c. will be measured by the arcs $N1$, $N2$, &c. Hence, in the right angled triangle $pN1$, $pN=$ the complement of the latitude, and the angle $Np1=15°$; therefore, (Trig. Art. 210) *rad.* : *tan.* 15° :: *sin.* pN : *tan.* N 1 ; in the same manner we find $N2$, $N3$, &c. Hence, for the same reason as for the horizontal dial, if $zabc$ be a plane coinciding with $NLzK$, and st be parallel to Op, st will project it's shadow in the same manner on the plane $zabc$, as Op does on the plane $NLzK$, and therefore the hour angles from the 12 o'clock line are computed by the same proportion. This is a *vertical south* dial. In the same manner the shadow may be projected upon a plane in any position, and the hour angles calculated.

(107.) In order to fix an horizontal dial, we must be able to tell the exact time of the sun's coming to the meridian; for which purpose, find the time (92) by the sun's altitude when it is at the solstices, that being the best time of the year for the purpose, because then the declination does not vary, and set a well-regulated watch to that time; then, when the watch shows 12 o'clock, the sun is on the meridian; at that instant, therefore, set the dial, so that the shadow of the gnomon may coincide with the 12 o'clock line, and it stands right.

(108.) Hence, we may easily draw a meridian line upon an horizontal plane. Suspend a plumb line so that the shadow of it may fall upon the plane, and when the watch shows 12, the shadow of the plumb line is the true meridian. The common way is to describe several concentric circles upon an horizontal plane, and in the center to erect a gnomon perpendicularly to it, with a small round well defined head, like the head of a pin; make a point upon any one of the circles where the shadow of the head falls upon it in the morning, and again where it falls upon the same circle in the afternoon; draw two radii from these two points, and bisect the angle between them, and the bisecting line will be a meridian line. This should be done when the sun is at the tropic, when it does not sensibly change it's declination in the interval of the observations; for if it do, the sun will not be equidistant from the meridian at equal altitudes. But this method is not capable of very great accuracy; for the shadow not being very accurately defined, it is not easy to say at what instant of time the shadow of the head of the gnomon is bisected by the circle. If, however, several circles be made use of, and the mean of the whole number of meridians so taken, be drawn, the meridian may be found with sufficient accuracy for all common purposes.

(109.) To find whether a wall be full south for a vertical south dial, erect a gnomon perpendicularly to it, and hang a plumb line from it; then when the watch, as above adjusted, shows 12, if the shadow of the gnomon coincide with the plumb line, the wall is full south.

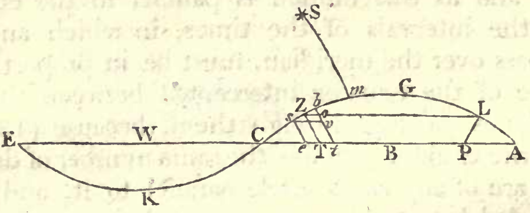
CHAP. III.

TO DETERMINE THE RIGHT ASCENSION, DECLINATION, LATITUDE AND LONGITUDE OF THE HEAVENLY BODIES.

(110.) THE foundation of all Astronomy is to determine the situation of the fixed stars, in order to find, by a reference to such fixed objects, the places of the other bodies at any given time, and thence to deduce their proper motions. The positions of the fixed stars are found from observation, by knowing their right ascensions and declinations (41); and these are found by means of the transit telescope and astronomical quadrant, as explained in my *Treatise on Practical Astronomy*; and then, by computation, their latitudes and longitudes may be found.

(111.) As the earth revolves uniformly about it's axis, the apparent motion of all the heavenly bodies, arising from this motion of the earth, must be uniform; and as this motion is parallel to the equator (76), the intervals of the times, in which any two stars pass over the meridian, must be in proportion to the arc of the equator intercepted between the two secondaries passing through them, because (13) this arc of the equator contains the same number of degrees as the arc of any small circle parallel to it, and comprehended between the same secondaries; and therefore, if one increase uniformly, the other must. Hence, the right ascension of stars passing the meridian at different times, will differ in proportion to the difference of the times of their passing, that is, if one star pass the meridian 1 hour before another, the difference of their right ascensions is 15°. Hence, if the clock be supposed to go uniformly, we have the following rule: *As the interval of the times of the succeeding passages of any one fixed star over the meridian : the interval of the passages of any two stars :: 360° : their difference of right ascensions**. By the same method we may find the difference of right ascensions of the sun or moon, when they pass the meridian, and a star, and therefore if that of the star be known, that of the sun or moon will; which conclusion will be more exact, if we compare them with several stars, and take the mean.

(112.) Now to determine the right ascension of a fixed star, Mr. *Flamstead* proposed a method, by comparing the right ascension of the star with that of the sun when near the equinoxes, the sun having the same declination each time; and as this method has not been noticed by any writers, we shall give an explanation. Let ***AGCKE*** be the equator, ***ABCWE*** the ecliptic, *S* the place of the star, *Sm* a secondary to the equator, and let the sun be at ***P***, near to ***A***, when it is on the meridian, and take ***CT=PA***, and draw ***PL***, ***TZ***, perpendicular to ***AGC***, and ***ZL*** is parallel to ***AC***, and the sun's declination is the same at ***T*** as at ***P***. Observe the meridian altitude of the sun when at ***P***, and also the time of the passage of it's center over the meridian; observe also at what time the star passes over the meridian, and then (111) find



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* A small correction must here be applied for the aberration of the star, in order to get the true difference of right ascensions, as will be explained; because there is a small difference between the true and apparent places.

the apparent difference *Lm* of their right ascensions. When the sun approaches near to *T*, observe it's meridian altitude for several days, so that on one of them, at *t*, it may be greater, and on the next day, at *e*, it may be less than the meridian altitude at *P*, so that in the intermediate time it must have passed through *T*; and drawing *tb*, *es*, perpendicular to *AGCE*, observe, on these two days, the differences *bm*, *sm* of the sun's right ascension and that of the star; draw also *sv* parallel to *Zo*. Then, to find *Zb*, we may consider the variation both of the right ascension and declination, to be uniform for a small time, and consequently to be proportional to each other; hence, *vb* (the change of meridian altitudes in one day) : *ob* (the difference of the meridian altitudes at *t* and *T*, or the difference of declinations) :: *sb* (the difference of *sm*, *bm* found by observation) : *Zb*, which added to *bm*, or subtracted from it, according to the situation of *m*, gives *Zm*, to which add *Lm*, or take their difference, according to circumstances, and we get *ZL*, which subtracted from *AGC*, or 180°, half the remainder will be *AL*, the sun's right ascension at the first observation, to which add *Lm*, and we get the star's right ascension at the same time. Instead of finding *bZ*, we might have found *sZ*, by taking $TZ - es$ for the second term, and thence we should have got *Zm*. Thus we should get the right ascension of a star, upon supposition that the position of the equator had remained the same, and the apparent place of the star had not varied in the interval of the observations. But the intersection of the equator with the ecliptic has a retrograde motion, called the Precession of the Equinoxes; also, the inclination of the equator to the ecliptic is subject to a variation, called the Nutation; and from the aberration of the star, it's apparent place is continually changing; these must therefore be allowed for, by considering how much they have varied in the interval of the observations; but these are not subjects to be treated of in an elementary treatise.

Having thus determined the right ascension of one star, that of the rest may be found from it (111).

(113.) The practical method of finding the right ascension of a body from that of a fixed star, by a clock adjusted to *sidereal* time*, is this: Let the clock begin it's motion from 0*h*. 0′. 0″ at the instant the first point of Aries is on the meridian; then, when any star comes to the meridian, the clock will show the apparent right ascension of the star, the right ascension being estimated in the time, at the rate of 15° for an hour, provided the clock is subject to no error, because it will then show, at any time, how far the first point of Aries is from the meridian. But as the clock is necessarily liable to err, we must be able, at any time, to ascertain what it's error is, that is, what is the difference between the right ascension shown by the clock, and the right ascension of that point of the equator which is at that time on the meridian. To do this, we must, when a star, whose apparent right ascension is known, passes the meridian, compare it's apparent right ascension with the right ascension shown by the clock, and the difference will show the error of the clock. For instance, let the apparent right ascension of *Aldebaran* be 4*h*. 23′. 50″ at the time when it's transit over the meridian is observed by the clock, and suppose the time shown by the clock to be 4*h*. 23′. 52″, then there is an error of 2″ in the clock, it giving the right ascension of the star 2″ more than it ought. If the clock be compared with several stars, and the mean error taken, we shall have, more accurately, the error at the mean time of all the observations. These observations being repeated every day, we shall get the rate of the clock's going, that is, how fast it gains or loses. The error of the clock, and the rate of it's going, being thus ascertained, if the time of the true transit of any body be observed, and the error of the clock at the time be applied, we shall have the right ascension of the body. This is the method by which the right ascension of the sun, moon, and planets are regularly found in observatories.

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* A clock is said to be adjusted to *sidereal* time, when it is adjusted to go 24 hours from the time a fixed star leaves the meridian till it returns to it, or it is the time of a revolution of the earth about it's axis.

(114.) The right ascension of the heavenly bodies being thus ascertained, the next thing to be explained is, the method of finding their declinations. Take the apparent altitude of the body, when it passes the meridian, by an astronomical quadrant, as explained in my *Treatise on Practical Astronomy*; correct it for parallax and refraction (Chap. VI. and VII.) and you get the true meridian altitude, *Ht*, or *He*, (Fig. page 25), the difference between which and the altitude *HE* of the equator (which, by Art. 87, is equal to the complement of the latitude previously determined) is the declination *Et*, or *Ee*, required.

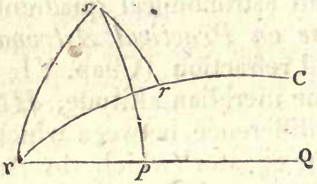
Ex. On April 27, 1774, the zenith distance of the moon's lower limb, when it passed the meridian at Greenwich, was 68°. 19′. 37″,3; it's parallax in altitude was 56′. 19″,2, allowing for the spheroidical figure of the earth; the barometer stood at 29, 58, and the thermometer at 49; to find the declination.

Observed zenith distance of L. L.	68°. 19′. 37″3
Refr. cor. for bar. and ther. -	+ 2. 23
	68. 22. 00,3
Parallax - - - - - - -	− 56. 19,2
True zenith distance of L. L. -	67. 25. 41,1
Semidiameter - - - - - -	− 16. 35
True zenith distance of the center	67. 9. 6,1
Latitude - - - - - - -	51. 28. 40
Declination *south* - - - - -	15. 40. 26,1

The *horizontal* parallax and semidiameter may be taken from the Nautical Almanac; and the parallax in altitude may be found, as will be explained when we come to treat of the Parallax.

Given the Right Ascension and Declination of an Heavenly Body, and the Obliquity of the Ecliptic, to find the Latitude and Longitude.

(115.) Let *s* be the body, ♈ *C* the ecliptic, ♈ *Q* the equator, *sr*, *sp* perpendicular to ♈ *C*, ♈ *Q*. Then (Trig. Art. 212) tan. *sp* : rad. :: sin. ♈ *p* : cot. *s* ♈ *p*. Hence, $s \text{♈} p + Q \text{♈} C = s \text{♈} r$. Also,



cos. *s* ♈ *p* : rad. :: tan. *p* ♈ : tan. *s* ♈ (Trig. Art. 219)
rad. : cos. *s* ♈ *r* :: tan. *s* ♈ : tan. *r* ♈ (Trig. Art. 219)

∴ cos. *s* ♈ *p* : cos. *s* ♈ *r* :: tan. *p* ♈ : tan. *r* ♈ = $\frac{\cos.\ s \text{♈} r \times \tan.\ p \text{♈}}{\cos.\ s \text{♈} p}$ the tangent of the *longitude*; and the logarithmic operation is,

ar.co.log.cos.s ♈ *p* + *log.cos.s* ♈ *r* + *log.tan.p* ♈ − 10, = *log.tan.r* ♈.

Also, (Trig. Art. 210) rad. : sin. *r* ♈ :: tan. *r* ♈ *s* : tan. *sr* the tangent of *latitude*; and the logarithmic operation is,

log. sin. r ♈ + *log. tan. r* ♈ *s* − 10, = *log. tan. sr.*

In this manner, the right ascensions and declinations of the fixed stars being found from observation, their latitudes and longitudes may be computed, and their places become determined (41); hence, a catalogue of the fixed stars may be made for any time.

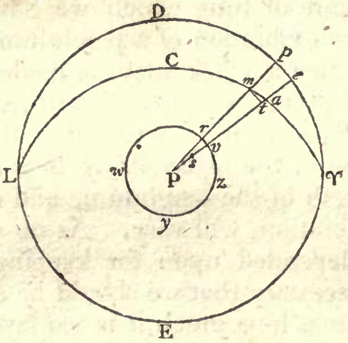
If the latitude and longitude be given, the right ascension and declination may be found in the same manner; considering ♈ *C* the equator, and ♈ *Q* the ecliptic.

CHAP. IV.

ON THE EQUATION OF TIME.

(116.) HAVING explained, in the last Chapter, the practical methods of determining the place of any body in the Heavens, we come next to the consideration of another circumstance not less important, that is, the irregularity of time as measured by the sun. The best measure of time which we have, is a clock regulated by the vibration of a pendulum. But with whatever accuracy a clock may be made, it must be subject to go irregularly, partly from the imperfection of the workmanship, and partly from the expansion and contraction of the materials by heat and cold, by which the length of the pendulum, and consequently the time of vibration, will vary. As no clock, therefore, can be depended upon for keeping time accurately, it is necessary that we should be able to ascertain, at any time, how much it is too fast or too slow, and at what rate it gains or loses. For this purpose, it must be compared with some motion which is uniform, or of which, if it be not uniform, you can ascertain the variation. The motions of the heavenly bodies have therefore been considered as most proper for this purpose. Now the earth revolving uniformly about it's axis, the apparent diurnal motion of the fixed stars about the axis must be uniform. If a clock, therefore, be adjusted to go 24 hours from the passage of any fixed star over the meridian till it returns to it again, it's rate of going may be at any time determined by comparing it with the transit of any fixed star, and observing whether the interval continues to be 24 hours; if not, the difference shows how much it gains or loses in that time. A clock adjusted to go 24 hours in this interval, is said to be adjusted to *sidereal* time. But if we compare a clock with the sun, and adjust it to go 24 hours from the time the sun leaves the meridian on any day, till he returns to it the next day, which is a *true* solar day, the clock will not, even if it go uniformly, continue to agree with the sun, that is, it will not show 12 when the sun comes to the meridian.

(117.) For let *P* be the pole of the earth, *vwyz* it's equator, and suppose the earth to revolve about it's axis in the order of the letters *vwyz*; let ♈ *DLE* be the celestial equator, and ♈ *CL* the ecliptic, in which the sun moves according to that direction. Let the sun be at *a* when it is upon the meridian of any place on any one day, and *m* the place when it is on the meridian the next day, and draw *Pvae*, *Prmp*, secondaries to the equator, and let the spectator be at *s* on the meridian *Pv*, with the sun at *a* on his meridian. Then when the earth has made one revolution about it's axis, *Psv* is come again into the same position; but the sun having moved forward, the earth must continue to revolve, in order to bring the meridian *Psv* into the position *Prm*, so that the sun at *m* may be again in the spectator's meridian. Now the angle *vPr* is measured by the arc *ep*, which is the increase of the sun's right ascension in a *true* solar day, the right ascension being measured upon the equator



♈ *DLE* (41); hence, the length of a *true* solar *day, is equal to the time of the earth's rotation about it's axis*+*the time of it's describing an angle equal to the increase of the sun's right ascension on a true solar day.* Now if the sun moved *uniformly*, and *in the equator* ♈ *DLE*, this increase, *ep*, would be always the same in the same time, and therefore the solar days would be always equal; but the sun moves in the ecliptic ♈ *CL*, and therefore, *if* it's motion were *uniform*, equal arcs (*am*) upon the ecliptic would not give equal arcs (*ep*) upon the equator*. But the motion of the sun is *not* uniform, and therefore *am*, described in any given time, is subject to a variation, which must, on this account also, necessarily make *ep* variable. Hence, the increase, *ep*, of the sun's right ascension in a day, varies from two causes, that is, from the ecliptic not coinciding with the equator, and from the unequal motion of the sun in the ecliptic; therefore the length of a true solar day is subject to a continual variation; consequently a clock, adjusted to go 24 hours for any one true solar day, will not continue to show 12 when the sun comes to the meridian, because the intervals by the clock will continue equal (the clock being supposed neither to gain nor lose), but the intervals of the sun's passage over the meridian will vary.

(118.) As the sun moves through 360° of right ascension in $365\frac{1}{4}$ days very nearly, therefore $365\frac{1}{4}$ days : 1 day :: 360°. : 59′. 8″,2 the increase of right ascension in one day, *if* the increase were uniform, in which case the solar days would be equal, and these days are called *mean* solar days. If therefore a clock be adjusted to go 24 hours in a *mean* solar day, it cannot continue to coincide with the sun, that is, to show 12 when the sun is on the meridian; but the sun will pass the meridian, sometimes *before* 12 and sometimes *after*. This difference is called the *Equation of Time*. A clock thus adjusted, is said to be adjusted to *mean solar time**. The time shown by the clock is called *true* or *mean* time, and that shown by the sun is called *apparent* time.

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* For draw *mt* parallel to *ep*, and suppose *ma* to be indefinitely small; then by plain trigonometry,

ma : *mt* :: rad. : sin. *mat*, or ♈ *ae* (Trig. Art. 125)
mt : *ep* :: cos. *ae* : rad. (Art. 13)

∴ *ma* : *ep* :: cos. *ae* : sin. ♈ *ae* :: (because sin. ♈ *ae* = $\frac{\text{cos. } a\text{♈}e \times \text{rad.}}{\text{cos. } ae}$ Trig. Art. 212) $\overline{\text{cos. } ae}^{2}$: cos. *a*♈*e*×radius : hence, the ratio of *ma* to *ep* is variable; if therefore the sun's motion *ma* were uniform, the corresponding increase *ep* of right ascension would not be uniform.

(119.) A clock adjusted to go 24 hours in a *mean* solar day, would coincide with an imaginary star moving uniformly in the equator with the sun's mean motion 59′. 8″,2 in right ascension, if the star were to set off from any given meridian when the clock shows 12; that is, the clock would always show 12 when the star came to the meridian, because the interval of the passages of this star over the meridian would be a *mean* solar day. This star, therefore, if we reckon it's motion from the meridian, in time, at the rate of 1 hour for 15°, would always coincide with the clock; that is, when the clock shows 1 hour, the star's motion would be 1 hour in right ascension, reckoned in time at the rate of 15° for an hour; when the clock shows 2 hours, the star's motion would be 2 hours; and so on. Hence, this star may be substituted instead of the clock; therefore, when the sun passes the given meridian, the difference between it's right ascension and that of the star, converted into time, is the difference between the time when the sun is on the meridian and 12 o'clock, or the equation of time; because the given meridian passes through the star at 12 o'clock, and it's motion, in respect to the star, is at the rate of 15° in an hour (121).

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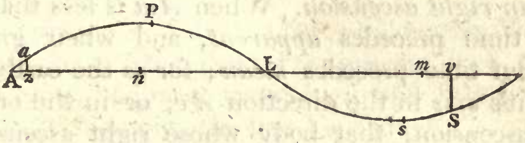
* As the earth describes an angle of 360°. 59′. 8″,2 about it's axis in a *mean* solar day of 24 hours, and an angle of 360° in a *sidereal* day, therefore 360°. 59′. 8″,2 : 360°. :: 24*h*. : 23h. 56′. 4″,098 the length of a sidereal day in mean solar time, or the time from the passage of a fixed star over the meridian, till it returns to it again.

(120.) Now, to compute the equation of time, let *APLS* be the ecliptic, *ALv* the equator, *A* the first point of aries, *P* the sun's apogee, *S* any place of the sun; draw *Sv* perpendicular to the equator, and take $An = AP$. When the sun departs from *P*, let the imaginary star set out from *n* with the sun's mean motion in right ascension, or in longitude, or at the rate of 59′. 8″,2 in a day, and when *n* passes the meridian, let the clock be adjusted to 12, as described in the last article: these are the corresponding positions of the clock and sun, as assumed by astronomers. Take $nm = Ps$, and when the star comes to *m*, the place of the sun, if it moved uniformly with it's mean motion, would be at *s*, but at that time let *S* be the place of the sun, and let the sun at *S*, and consequently *v*, be on the meridian; then as *m* is the place of the imaginary star at that instant, *mv* is the equation of time. Let *a* be the *mean* equinox*, and draw *az* perpendicular to *AL*; then *z* on the equator would have coincided with *a*, if the equinox had moved uniformly; therefore we must reckon the *mean* right ascension from *z*. Now $mv = Av - Am$; but $Am = Az + zm$ $= Aa \times \cos.\, aAz + zm =$ (because $\cos.\, aAz$ (23°. 28′) $= \frac{11}{12}$ very nearly) $\frac{11}{12} Aa + zm$; hence, $mv = Av - zm$



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* The equinox has a retrograde motion, and that motion is not uniform; we here therefore suppose *a* to be the point where the equinox would have been, if it moved uniformly with it's *mean* velocity.

$-\frac{11}{12} Aa$; but Av is the sun's true right ascension, zm is the mean right ascension, or mean longitude, and $\frac{11}{12} Aa$ (Az) is the equation of the equinoxes in right ascension; hence, the equation of time is equal to *the difference of the sun's true right ascension, and it's mean longitude corrected by the equation of the equinoxes in right ascension.* When Am is less than Av, *mean* time precedes *apparent,* and when *greater, apparent* time precedes *mean*; for as the earth turns about it's axis in the direction Av, or in the order of right ascension, that body whose right ascension is least must come upon the meridian first; that is, when the sun's true right ascension is *greater* than it's mean longitude corrected as above, we must *add* the equation of time to the apparent, to get the mean time; and when it is *less,* we must *subtract.* To convert mean time into apparent, we must *subtract* in the *former* case, and *add* in the *latter.* This rule for computing the equation of time, was first given by Dr. *Maskelyne* in the *Phil. Trans.* 1764.

(121.) As a meridian of the earth, when it leaves *m,* returns to it again in 24 hours, it may be considered, when it leaves that point, as approaching a point at that time 360° from it, and at which time it arrives in 24 hours. Hence, the relative velocity with which a meridian accedes to or recedes from *m,* is at the rate of 15° in an hour. Therefore, when the meridian passes through *v,* the arc *vm,* reduced into time at the rate of 15° in an hour, gives the equation of time at that instant. Hence, the equation of time is computed for the instant of *apparent* noon, or when the sun is on the meridian. Now the time of apparent noon in mean solar time, for which we compute, can only be known by knowing the equation of time. To compute, therefore, the equation on any day, you must assume the equation the same as on that day four years before, from which it will differ but very little, and it will give the time of apparent noon, sufficiently accurate for the purpose of computing the equation. If you do not know the equation four years before, compute the equation for noon mean time, and that will give apparent noon accurately enough.

Ex. To find the equation of time on July 1, 1792, for the meridian of Greenwich, by *Mayer's* Tables.

The equation on July 1, 1788, was, by the *Nautical Almanac*, 3′. 28″, to be added to apparent noon, to give the corresponding mean time; hence, for July 1, 1792, at 0*h*. 3′. 2″, compute the true longitude*.

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* The reason of this operation will appear to those who understand the method of computing the place of the sun from the solar tables. The explanation of such matters comes not within the plan of this work. See my *Complete System of Astronomy*.

	Mean Lon. ⊙	Long. ⊙'s Apog.	N°. 1.	N°. 2.	N°. 3.	N°. 4.
Epoch for 1792.	9^s. 10°. 50′. 0″,7	3^s. 9°. 23′. 46″	241	227	123	478
Mean Mot. July 1,	5. 29. 23. 16,2	33	163	456	312	27
3″	7,4					
28″	1,1					
Mean Longitude	3. 10. 13. 25,4	3. 9. 24. 19	404	683	435	505
Equat. of Center	− 1. 37,1	3. 10. 13. 25,4				
Equat. ☽ I.	+ 4,5		Mean Anomaly.			
♃ II.	− 4,7	49. 6,4				
♀ III.	+ 3,65					
☊ IV.	− 0,6					
True Longitude	3. 10. 11. 51,15					

With this true longitude, and obliquity 23°. 27′. 48″,4 of the ecliptic, the true right ascension of the sun is found to be $3^s. 11^\circ. 5'. 41'',25$; also, the equation of the equinoxes in longitude $= -0'',6$; hence,

The mean longitude - -	$3^s. 10^\circ. 13'. 25'',4$
$\frac{11}{12}$ of $-0'',6$ - - - -	$-\ 0,55$
Mean longitude corrected	3. 10. 13. 24,85
True right ascension - -	3. 11. 5. 41,25
Equation - - - - -	52. 16,4

Which converted into time, gives $3'. 29'',1$ for the true equation of time; which must be added to apparent, to give the true time, because the true right ascension is greater than the mean longitude.

(122.) The sun's apogee, *P*, has a progressive motion, and the equinoctial points, *A*, *L*, have a regressive motion; the inclination also of the equator to the ecliptic is subject to a constant variation. Hence, the same Table of the equation of time cannot continue to serve for the same degree of the sun's longitude. Also, the sun's longitude at noon at the same place is different for the same days on different years, and it is for apparent noon that the equation is computed. For these reasons, the equation of time must be computed a-new for every year.

(123.) The two inequalities are sometimes separately considered, thus: *First*, that arising from the obliquity of the ecliptic. Let the sun and the imaginary star set off together from *L*, and let us now assume $LS = Lm$; and let each move uniformly with the mean velocity, and then they will come to *S* and *m* together. Now when *LS* is greater than 90°, the hypothenuse, *LS*, is less than the base, *Lv* (Trig. Art. 197); therefore *Lm* is less than *Lv* (the case represented by the Figure); the star therefore, being left behind, comes upon the meridian first, and consequently true time precedes apparent. But when *LS* is less than 90°, the hypothenuse, *LS*, is greater than the base, *Lv*; therefore *Lm* is greater than *Lv*, and *m* lies on the other side of *v*; therefore the sun comes upon the meridian first; consequently apparent time precedes true. Hence, from equinox to tropic, apparent time precedes true; and from tropic to equinox, true time precedes apparent. *Secondly*, that arising from the unequal motion of the earth in it's orbit. Let us suppose the sun to move about the earth, instead of the earth about the sun, the effect here being just the same, and this supposition will render the explanation easier. Let the sun depart from the apogee, and let the imaginary star set off from thence at the same time, with the *mean* angular velocity of the sun. Now when the sun is at it's greatest distance, it's angular velocity it less than it's *mean* angular velocity, (Note to Art. 165), and consequently less than the velocity of the star; the star therefore gettting forwarder than the sun, the sun comes upon the meridian first, as shown in Art. 120, and therefore apparent time precedes true; and this will continue till the sun comes to it's least distance, where, having performed half it's revolution, and the star also having performed half it's revolution, the sun and star will coincide, (see Art. 168). Hence, from apogee to perigee, apparent time precedes true. Now the sun and star departing together from perigee, the sun's velocity is greater than that of the star; the star therefore being left behind, comes upon the meridian first, and true time precedes apparent; and this will continue till the sun comes to the apogee, where they again coincide. Hence, from perigee to apogee, true time precedes apparent.

(124.) Whenever the time is computed from the sun's altitude, that time must be *apparent* time, because we compute it from the time when the sun comes to the meridian, which is noon, or 12 o'clock apparent time, and will differ from the time shown by a well-regulated watch or clock, by the equation of time. A clock or watch may therefore be regulated by a good dial; for if you apply the equation, as before directed, to the apparent time shown by the dial, it will give the *mean* time, or that which the clock or watch ought to show.

(125.) The equation of time was known to, and made use of by, *Ptolemy*. *Tycho* employed only one part, that which arises from the unequal motion of the sun in the ecliptic; but *Kepler* made use of both parts. He further suspected, that there was a third cause of the inequality of solar days, arising from the unequal motion of the earth about it's axis. But the equation of time, as now computed, was not generally adopted till 1672, when *Flamstead* published a dissertation upon it, at the end of the works of *Horrox*.

CHAP. V.

ON THE LENGTH OF THE YEAR, THE PRECESSION OF THE EQUINOXES FROM OBSERVATION, AND OBLIQUITY OF THE ECLIPTIC.

(126.) FROM comparing the sun's right ascension every day with that of the fixed stars lying to the east and west, the sun is found constantly to recede from those on the west, and approach to those on the east; hence, it's apparent annual motion is found to be from west to east; and the interval of time from it's leaving any fixed star till it returns to it again, is called a *sidereal* year, being the time in which the sun completes it's revolution amongst the fixed stars, or in the ecliptic. But the sun, after it leaves either of the equinoctial points, returns to it again in a less time than it returns to the same fixed star, and this interval is called a *solar* or *tropical* year, because the time from it's leaving one equinox till it returns to it, is the same as from one tropic till it comes to the same again. This is the year on which the return of the season depends.

On the Sidereal Year.

(127.) To find the length of a *sidereal* year, On any day when the sun is at Z on the meridian, (Fig. page 42), take the difference, Zm, between the sun's right ascension when it passes the meridian, and that of a fixed star, S; and when the sun returns to the same part of the heavens the next year, compare it's right ascension with that of the same star for two days, one when their difference, bm, of right ascensions is less, and the other when the difference, *sm*, is greater than the difference, *Zm*, before observed; then *bs* is the increase of the sun's right ascension in the time, *t*; and as the increase of right ascension may be considered as uniform for a small time, we have *bs* : *bZ* :: *t* : the time, *T*, in which the right ascension is increased from *b* to *Z*; this time, *T*, therefore, added to the time of the observed right ascension at *b*, gives the time when the sun is at the same distance, *Zm*, in right ascension from the star, which it was when observed at *Z* the year before; the interval of these times is therefore the length of a sidereal year. The best time for these observations is about March 25, June 20, September 17, December 20, the sun's motion in right ascension being then uniform. Instead of observing the difference of the right ascensions, you may observe that of their longitudes.

If, instead of repeating the second observations the year after, there be an interval of several years, and you divide the observed interval of time when the difference of their right ascensions was found to be equal, by the number of years, you will have the length of a sidereal year more exactly.

Ex. On April 1, 1669, at 0*h*. 3′. 47″, mean solar time, M. *Picard* observed the difference between the sun's longitude and that of *Procyon* to be 3^{s}. 8°. 59′. 36″, which is the most ancient observation of this kind, the accuracy of which can be depended upon; see *Hist. Celeste, par M. le Monnier*, page 37. And on April 2, 1745, M. *de la Caille* found, by taking their difference of longitudes on the 2d and 3d, that at 11*h*. 10′. 45″, mean solar time, the difference of their longitudes was the same as at the first observation. Now as the sun's revolution was known to be nearly 365 days, it is manifest that it had made seventy-six complete revolutions, in respect to the same fixed star, in the space of 76 years 1*d*. 11*h*. 6′. 58″. Now in these 76 years, there were 58 of 365 days, and 18 bissextiles of 366 days; that interval therefore contains

27759*d*. 11*h*. 6′. 58″; which being divided by 76, the quotient is 365*d*. 6*h*. 8′. 47″, the length of a sidereal year. From the most accurate observations, the length of a sidereal year is found to be 365*d*. 6*h*. 6′. 11″,5.

On the Tropical Year.

(128.) Observe the meridian altitude, a, of the sun on the day nearest to the equinox; then the next year take it's meridian altitude on two following days, one when it's altitude, m, is less than a, and the next when it's altitude, n, is greater than a, then $n - m$ is the increase of the sun's declination in 24 hours; also, when the declination has increased by the quantity $a - m$, from the time when the meridian altitude m, was observed, the declination will then become a; and as we may consider the increase of declination to be uniform for a day, we have $n - m : a - m :: 24$ hours : the interval from the time when the sun was on the meridian on the first of the two days, till the sun has the same declination a, as at the observation the year before; and this time, added to the time when the sun's altitude m was observed, gives the time when the sun's place in the ecliptic had the same situation in respect to the equinoctial points, which it had at the time of the observation the preceding year; and the interval of these times is the length of a tropical year.

If instead of repeating the second observation the next year, there be an interval of several years, and you divide the interval between the times when the declination was found to be the same, by the number of years, you will get the tropical year more exactly.

Ex. M. *Cassini* informs us, that on March 20, 1672, his father observed the meridian altitude of the sun's upper limb at the Royal Observatory at Paris, to be 41°. 43′; and on March 20, 1716, he himself observed the meridian altitude of the upper limb to be 41°. 27′. 10″; and on the 21st to be 41°. 51′: therefore the difference of the two latter altitudes was 23′. 50″, and of the two former 15′. 50″; hence, 23′. 50″ : 15′. 50″ :: 24 hours : 15*h*. 56′. 39″; therefore, on March 20, 1716, at 15*h*. 56′. 39″, the sun's declination was the same as on March 20, 1672. Now the interval between these two observations was 44 years, of which 34 consisted of 365 days each, and 10 of 366; therefore the interval in days was 16070; hence, the whole interval between the equal declinations was 16070 days 15*h*. 56′. 39″, which divided by 44, gives 365*d*. 5*h*. 49′. 0″. 53‴, the length of a tropical year from these observations. From the best observations, the length of a tropical year is found to be 365*d*. 5*h*. 48′. 48″.

To find the Precession of the Equinoxes from Observation.

(129.) The sun returning to the equinox every year before it returns to the same point in the heavens, shows that the equinoctial points have a retrograde motion, and this arises from the motion of the equator, which is caused by the attraction of the sun and moon upon the earth, in consequence of it's spheroidical figure. The effect of this is, that the longitude of the stars must constantly increase; and by comparing the longitude of the same stars at different times, the motion of the equinoctial points, or the precession of the equinoxes, may be found.

(130.) *Hipparchus* was the first person who observed this motion, by comparing his own observations with those which *Timocharis* made 155 years before. From this he judged the motion to be one degree in about 100 years; but he doubted whether the observations of *Timocharis* were accurate enough to deduce any conclusion to be depended upon. In the year 128 before J. C. he found the longitude of *Virgin's Spike* to be 5^s. 24°; and in the year 1750 it's longitude was found to be 6^s. 20°. 21′, the difference of which is 26°. 21′. In the same year he found the longitude of the *Lyon's Heart* to be 3^s. 29°. 50′; and in 1750 it was 4^s. 26°. 21′, the difference of which is 26°. 31′. The mean of these two gives 26°. 26′ for the increase of longitude in 1878 years, or 50″. 40‴ in a year, for the precession. By comparing the observations of *Albategnius*, in the year 878, with those made in 1738, the precession appears to be 51″. 9‴. From a comparison of 15 observations of *Tycho*, with as many made by M. *de la Caille*, the precession is found to be 50″. 20‴. But M. *de la Lande*, from the observations of M. *de la Caille*, compared with those in *Flamstead*'s Catalogue, determines the secular precession to be 1°. 23′. 45″, or 50″,25 in a year.

(131.) The precession being given, and also the length of a tropical year, the length of a sidereal year may be found by this proportion; 360° – 50″,25 : 360°. :: 365*d.* 5*h.* 48′. 48″ : 365*d.* 6*h.* 9′. 11″$\frac{1}{2}$ the length of the *sidereal* year.

On the Anomalistic Year.

(132.) The year, called the *anomalistic* year, is sometimes used by astronomers, and is the time from the sun's leaving it's apogee, till it returns to it. Now the progressive motion of the apogee in a year is 11″,75, and hence, the anomalistic must be longer than the sidereal year, by the time the sun takes in moving over 11″,75 of longitude at it's apogee; but when the sun is in it's apogee, it's motion in longitude is 58′. 13″ in 24 hours; hence, 58′. 13″ : 11″,75 :: 24 hours : 4′. 50″$\frac{2}{3}$, which added to 365*d.* 6*h.* 9′. 11$\frac{1}{2}$″, gives 365*d.* 6*d.* 14′. 2″$\frac{1}{6}$ the length of the anomalistic year. M. *de la Lande* determined this motion of the apogee, from the observations of M. *de la Hire* and those of Dr. *Maskelyne*. *Cassini* made it the same.

On the Obliquity of the Ecliptic.

(133.) The method used by astronomers to determine the obliquity of the ecliptic, is that explained in Art. 86, by taking half the difference of the greatest and least meridian altitudes of the sun. The following is the obliquity, as determined by different astronomers.

Eratosthenes 230 years before *J. C.*	23°.	51′.	20″
Hipparchus 140 years before *J. C.*	23.	51.	20
Ptolemy 140 years after *J. C.*	23.	51.	10
Pappus in the year 390	23.	30.	0
Albategnius in 880	23.	35.	40
Arzachel in 1070	23.	34.	0
Prophatius in 1300	23.	32.	0
Regiomontanus in 1460	23.	30.	0
Waltherus in 1490	23.	29.	47
Copernicus in 1500	23.	28.	24
Tycho in 1587	23.	29.	30
Cassini (the Father) in 1656	23.	29.	2
Cassini (the Son) in 1672	23.	28.	54
Flamstead in 1690	23.	28.	48
De la Caille in 1750	23.	28.	19
Dr. *Bradley* in 1750	23.	28.	18
Mayer in 1750	23.	28.	18
Dr. *Maskelyne* in 1769	23.	28.	8,5
M. *de la Lande* in 1768	23.	28.	0

The observations of *Albategnius,* an Arabian, are here corrected for refraction. Those of *Waltherus,* M. *de Caille* computed. The obliquity by *Tycho* is here put down as correctly computed from his observations. Also, the obliquity, as determined by *Flamstead,* is corrected for the nutation of the earth's axis. These corrections M. *de la Lande* applied.

(134.) It is manifest, from the above observations, that the obliquity of the ecliptic continually decreases;

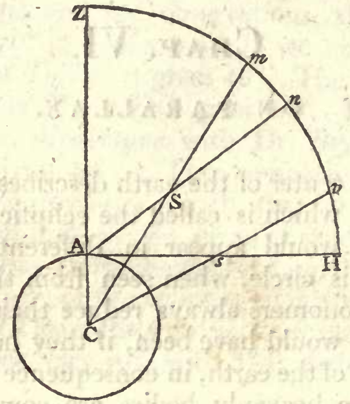
and the irregularity which here appears in the diminution we may ascribe to the inaccuracy of the observations, as we know that they are subject to greater errors than the irregularity of this variation. If we compare the first and last observations, they give a diminution of 70″ in 100 years. If we compare the last with that of *Tycho*, it gives 45″. The last, compared with that of *Flamstead*, gives 50″. If we compare that of Dr. *Maskelyne* with Dr. *Bradley's* and *Mayer's*, it gives 50′. The comparison of Dr. *Maskelyne's* determination, with that of M. *de la Lande*, which he took as the mean of several results, gives 50″. We may therefore state the secular diminution of the obliquity of the ecliptic, at this time, to be 50″, as determined from the most accurate observations. This result agrees very well with that deduced from theory.

CHAP. VI.

ON PARALLAX.

(135.) THE center of the earth describes that circle in the heavens which is called the ecliptic; but as the same object would appear in different positions in respect to this circle, when seen from the center and surface, astronomers always reduce their observations to what they would have been, if they had been made at the center of the earth, in consequence of which, the places of the heavenly bodies are computed as seen from the ecliptic, and it becomes a fixed plane for that purpose, on whatever part of the earth's surface the observations are made.

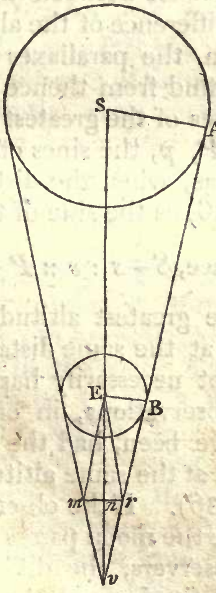
(136.) Let C be the center of the earth, A the place of the spectator on it's surface, S any object, ZH the sphere of the fixed stars, to which the places of all the bodies in our system are referred; Z the zenith, H the horizon; draw CSm, ASn, and m is the place of S seen from the center, and n from the surface. Now the plane SAC passing through the center of the earth, must be perpendicular to it's surface, and consequently it will pass through the zenith Z; and the points, m, n, lying in the same plane, the arc of parallax, mn, must lie in a circle perpendicular to the horizon; and hence, the azimuth is not affected, if the earth be a sphere. Now the parallax, mn, is measured by the angle mSn, or ASC; and (Trig. Art. 128) $CS : CA :: \sin. SAC$, or $\sin. SAZ$, $: \sin. ASC$ the parallax $= \frac{CA \times \sin. SAZ}{CS}$. As CA is constant, supposing the earth to be a sphere, the sine of the parallax varies as the sine of the apparent zenith distance directly, and the distance of the body from the center of the earth inversely. Hence, a body in the zenith has no parallax, and at s in the horizon it is the greatest. And



if the object be at an indefinitely great distance, it has no parallax; hence, the apparent places of the fixed stars are not altered by it. As n is the apparent place, and m is called the true place, the parallax depresses an object in a vertical circle. For different altitudes of the same body, the parallax varies as the sine, s, of the apparent zenith distance; therefore, if $p =$ the horizontal parallax, and radius be unity, we have $1 : s :: p : ps$, the sine of the parallax. To ascertain, therefore, the parallax at all altitudes, we must first find it at some given altitude.

(137.) *First* method, for the *sun.* *Aristarchus* proposed to find the sun's parallax, by observing it's elongation from the moon at the instant it is dichotomized, at which time the angle at the moon is a right angle; therefore we may find the angle which the distance of the moon subtends at the sun; which, diminished in the ratio of the moon's distance from the earth's center to the radius of the earth, would give the sun's horizontal parallax. But a very small error in the time when the moon is dichotomized (and it is impossible to be very accurate in this), will make so very great an error in the sun's parallax, that nothing can be deduced from it to be depended upon. *Vendelinus* determined the angle of elongation when the moon was dichotomized, to be 89°. 45′, from which the sun's parallax was found to be 15″. But *P. Riccioli* found it to be 28″ or 30″ from like observations.

(158.) *Second* method. *Hipparchus* proposed to find the sun's parallax from a lunar eclipse, by the following method. Let S be the sun, E the earth, Ev the length of it's shadow, mr the path of the moon in a central eclipse. Observe the length of this eclipse, and then, from knowing the periodic time of the moon, the angle mEr, and consequently the half of that angle, or nEr, will be known. Now the horizontal parallax, ErB, of the moon being known, we have the angle $Evr = ErB - nEr$; hence, we know EAB* $= AES - Evr = AES - ErB + nEr$; that is, the sun's horizontal parallax = the apparent semidiameter of the



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* Supply the line AE.

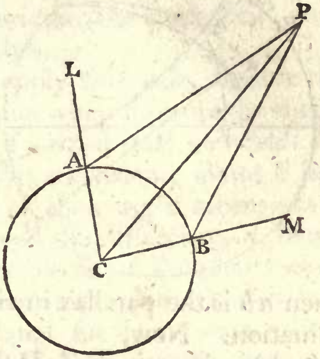
sun − the horizontal parallax of the moon + the semidiameter of the earth's shadow where the moon passes through it. The objection to this method is, the great difficulty of determining the angle nEr, with sufficient accuracy; for any error in that angle will make the same error in the sun's parallax, the other quantities remaining the same. By this method, *Ptolemy* made the sun's horizontal parallax $2'.\ 50''$. *Tycho* made it $3'$.

(139.) *Third* method, for the *moon*. Take the meridian altitudes of the moon, when it is at it's greatest north and south latitudes, and correct them for refraction; then the difference of the altitudes, thus corrected, would be equal to the sum of the two latitudes of the moon, if there were no parallax; consequently the difference between the sum of the two latitudes and the difference of the altitudes, will be the difference between the parallaxes at the two altitudes. Now, to find from thence the parallax itself, let S, s, be the sines of the greatest and least apparent zenith distances, P, p, the sines of the corresponding parallaxes; then as, when the distance is given, the parallax varies (136) as the sine of the zenith distance,

$$S : s :: P : p; \text{ hence, } S-s : s :: P-p : p = \frac{s \times \overline{P-p}}{S-s}$$

the parallax at the greatest altitude. This supposes that the moon is at the same distance in both cases; but as this will not necessarily happen, we must correct one of the observations, in order to reduce it to what it would have been, had the distance been the same, the parallax at the same altitude being inversely as the distance (136). If the observations be made in those places where the moon passes through the zenith of one of the observers, the difference between the sum of the two latitudes and the zenith distance at the other observation, will be the parallax at that altitude.

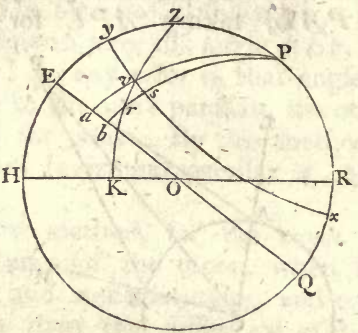
(140.) *Fourth* method. Let a body, P, be observed from two places, A, B, in the same meridian, then the whole angle APB, is the sum of the two parallaxes of the two places. The parallax (136) $APC = hor.\ par. \times \sin.\ PAL$, taking APC for $\sin.\ APC$,



and the parallax $BPC = hor.\ par. \times \sin.\ PBM$; hence, $hor.\ par. \times \overline{\sin.\ PAL + \sin.\ PBM} = APB$, $\therefore hor.\ par. = APB$ divided by the sum of these two sines. If the two places be not in the same meridian, it does not signify, provided we know how much the altitude varies from the change of declination of the body in the interval of the passages over the meridians of the two observers.

Ex. On Oct. 5, 1751, M. *de la Caille*, at the Cape of Good Hope, observed *Mars* to be 1′. 25″,8 below the parallel of λ in Aquarius, and at 25° distance from the zenith. On the same day, at Stockholm, Mars was observed to be 1′. 57″,7 below the parallel of λ, and at 68°. 14′ zenith distance. Hence, the angle APB is 31″,9, and the sines of the zenith distances being 0,4226 and 0,9287, the horizontal parallax was 23″,6. Hence, if the ratio of the distance of the earth from Mars to it's distance from the sun be found, we shall have the sun's horizontal parallax, the horizontal parallaxes being inversely as their distances from the earth (136).

(141.) *Fifth* method. Let EQ be the equator, P it's pole, Z the zenith, v the true place of the body, and r the apparent place, as depressed by parallax in the vertical circle ZK, and draw the secondaries,



Pva, Prb; then ab is the parallax in right ascension, and rs in declination. Now,

$vr : vs :: 1$ (rad,) : sin. vrs, or ZvP (Trig. Art. 125)
$\therefore vs : ab ::$ cos. $va : 1$ (rad.) Art. 13.

$\therefore vr : ab ::$ cos. va : sin. ZvP;

therefore $ab = \frac{vr \times \sin. ZvP}{\cos. va}$; but $vr =$ *hor. par.* × sin. vZ (136), and (Trig. Art. 221) sin. vZ : sin. ZP :: sin. ZPv : sin. $ZvP = \frac{\sin. ZP \times \sin. ZPv}{\sin. vZ}$; therefore, by substitution, $ab = \frac{hor. par. \times \sin. ZP \times \sin. ZPv}{\cos. va}$. Hence, for the same star, where the *hor. par.* is given, the parallax in right ascension varies as the sine of the hour angle. Also, the *hor. par.* $= \frac{ab \times \cos. va}{\sin. ZP \times \sin. ZPv}$. For the eastern hemisphere, the apparent place b, lies on the equator to the east of a, the true place, and therefore the right ascension is increased by parallax; but in the western hemisphere, b lies to the west of a, and therefore the right ascension is diminished. Hence, if the right ascension be taken before and after the meridian, the whole change of parallax in right ascension between the two observations, is the sum (s) of the two parts before and after the meridian, and the

$hor.\ par. = \frac{s \times \cos.\ va}{\sin.\ ZP \times S}$. On the meridian there is no parallax in right ascension, for $ab = \frac{vr \times \sin.\ ZvP}{\cos.\ va}$, and on the meridian, the angle ZvP, and consequently it's sine, vanishes.

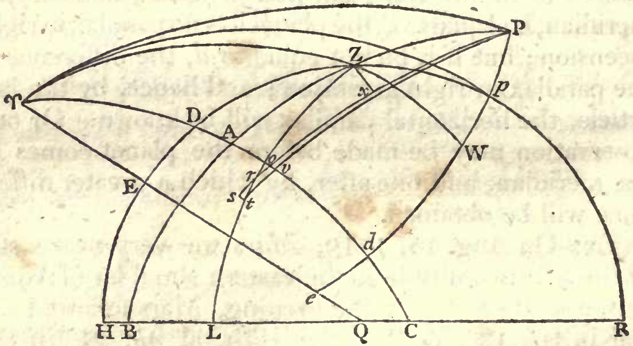
(142.) To apply this rule, observe the right ascension of the planet when it passes the meridian, compared with that of a fixed star, at which time there is no parallax in right ascension; about 6 hours after, take the difference of their right ascensions again, and observe how much the difference, *d*, between the apparent right ascensions of the planet and fixed star has changed in that time. Next, observe the right ascension of the planet for 3 or 4 days when it passes the meridian, in order to get it's true motion in right ascension; then, if it's motion in right ascension in the above interval of time, between the taking of the right ascensions of the fixed star and planet on and off the meridian, be equal to *d*, the planet has no parallax in right ascension; but if it be not equal to *d*, the difference is the parallax in right ascension; and hence, by the last article, the horizontal parallax will be known. Or one observation may be made before the planet comes to the meridian, and one after, by which a greater difference will be obtained.

Ex. On Aug. 15, 1719, *Mars* was very near a star of the 5th magnitude in the eastern shoulder of Aquarius, and at 9*h*. 18′ in the evening, Mars followed the star in 10′. 17″, and on the 16th, at 4*h*. 21′ in the morning, it followed it in 10′. 1″, therefore, in that interval, the apparent right ascension of Mars had increased 16″ in time. But, according to observations made in the meridian for several days after, it appeared that Mars approached the star only 14″ in that time, from it's motion; therefore 2″ in time, or 30″ in motion, is the effect of parallax in the interval of the observations. Now the declination of Mars was 15°, the co-latitude 41°. 10′, and the two hour angles

49°. 15′, and 56°. 39′; therefore the *hor. par.* = $\frac{30'' \times \cos. 15°}{\sin. 41°.\,10' \times \sin. 49°.\,15' + \sin. 56°.\,39'} = 27\frac{1}{2}''$. But at that time, the distance of the earth from Mars was to it's distance from the sun, as 37 to 100, and therefore the sun's horizontal parallax comes out 10″,17.

(143.) But, besides the effect of parallax in right ascension and declination, it is manifest that the latitude and longitude of the moon and planets must also be affected by it; and as the determination of this, in respect to the moon, is in many cases, particularly in solar eclipses, of great importance, we shall proceed to show how to compute it, supposing that we have given the latitude of the place, the time, and consequently the sun's right ascension, the moon's true latitude and longitude, with her horizontal parallax.

(144.) Let *HZR* be the meridian, ♈ *EQ* the equator, *p* it's pole; ♈ *C* the ecliptic, *P* it's pole; ♈ the first point of Aries, *HQR* the horizon, *Z* the zenith, *ZL* a secondary to the horizon passing through the true place *r*, and apparent place *t*, of the moon; draw *Pt*, *Pr*, which produce to *s*, drawing the small circle *ts*, parallel to *ov*; then *rs* is the parallax in *latitude*, and *ov* the parallax in *longitude*. Draw the great circles, ♈ *P*, *PZAB*, *Ppde*, and *ZW*, perpendicular to *pe*; then, as ♈ *P* = 90°, and ♈ *p* = 90°, ♈ must (4) be the pole of *Ppde*, and therefore *d*♈ = 90°; consequently *d* is one of the solstitial points, ♋ or ♑; also, draw *Zx* per-



pendicular to Pr, and join Z♈, p♈. Now ♈ E, or the angle ♈ pE, or Zp♈, is the right ascension of the mid-heaven, which is known; $PZ = AB$ (because AZ is the complement of both) the altitude of the highest point, A, of the ecliptic above the horizon, called the nonagesimal degree, and ♈ A, or the angle ♈ PA, is it's longitude; also, Zp is the co-latitude of the place, and the angle ZpW is the difference between the right ascension of the mid-heaven ♈ pE, and ♈ e. Now, in the right angled triangle ZpW, (Trig. Art. 212) rad. × cos. $p =$ tan. $pW \times$ cot. pZ; therefore,

$$\log.\ \tan.\ pW = 10, + \log.\ \cos.\ p - \log.\ \cot.\ pZ, \text{ (Trig. Art. 213);}$$

hence, $PW = pW \pm pP$, where the *upper* sign takes place when the right ascension of the mid-heaven is *less* than 180°, and the *lower* sign, when *greater*. Also, in the triangles WZp, WZP, we have (Trig. Art. 231) sin. Wp : sin. WP :: tan. WPZ : tan. Wpz :: (Trig. Art. 82) cot. WpZ : cot. WPZ, or, tan. AP♈; therefore (Trig. Art. 49),

$$\text{Log.}\ \tan.\ AP♈ = \text{ar. co. lo.}\ \sin.\ Wp + \log.\ \sin.\ WP + \text{lo.}\ \cot.\ WpZ - 10;$$

and as we know ♈ o, or ♈ Po, the true longitude of the moon, we know APo, or ZPx. Also, in the triangle WPZ, we have (Trig. Art. 219) cos. WPZ, or sin. AP♈, : rad. :: tan. WP : tan. ZP; therefore,

$$\text{Log.}\ \tan.\ ZP = 10, + \log.\ \tan.\ WP - \log.\ \sin.\ AP♈.$$

Again, in the triangle ZPr, we know ZP, Pr, and the angle, P, from which the angle ZrP, or trs, may be thus found. In the right angled triangle ZPx, we know ZP, and the angle P; hence, (Trig. Art. 212) rad. × cos. $ZPx =$ cot. $PZ \times$ tan. Px; therefore,

$$\text{Log.}\ \tan.\ Px = 10, + \cos.\ ZPx - \log.\ \cot.\ PZ;$$

therefore we know rx; hence, (Trig. Art. 231) sin. rx : sin. Px :: tan. ZPx : tan. Zrx, or trs; therefore,

$$\text{Log.}\ \tan.\ Zrx = \text{ar. co.}\ \log.\ \sin.\ rx + \log.\ \sin.\ Px + \log.\ \tan.\ ZPx - 10;$$

also, in the right angled triangle Zrx, we have (Trig.

Art. 212) rad. × cos. Zrx = cot. Zr × tan. rx; therefore,

$$Log.\ cot.\ Zr = 10, + log.\ cos.\ Zrx - log.\ tan.\ rx.$$

With this *true* zenith distance Zr, find (136) the parallax, as if it were the *apparent* zenith distance, and it will give you the true parallax *nearly*; add this therefore to the *true* zenith distance, and you will get *nearly* the *apparent* zenith distance, to which compute again (136) the parallax, and thus you will get the true parallax, rt, extremely nearly; then, in the right angled triangle rst, which may be considered as plane, we have (Trig. Art. 125) rad. : cos. r :: rt : rs, the parallax in *latitude*; therefore,

$$log.\ rs = log.\ rt + log.\ cos.\ r - 10, = log.\ par.\ latitude.$$

Also, rad. : sin. r :: rt : ts; therefore,

$$log.\ ts = log.\ rt + log.\ sin.\ r - 10;$$

hence, (13) cos. tv : rad. :: ts : ov, the parallax in *longitude*; therefore,

$$log.\ ov = 10, + log.\ ts - log.\ cos.\ tv = log.\ par.\ longitude.$$

Ex. On January 1, 1771, at 9*h*. apparent time, in lat. 53° N. the moon's true longitude was 3^s. 18°. 27′. 35″, and latitude 4°. 5′. 30″ S. and it's horizontal parallax 61′. 9″; to find it's parallax in latitude and longitude.

The sun's right ascension was 282°. 22′. 2″ by the Tables, and it's distance from the meridian 135°; also, the right ascension ♈ *E*, of the mid-heaven, was 57°. 22′. 2″; hence, the whole operation for the solution of the triangles will stand thus:

Tri. ZpW	ZpW	= 32°. 37′. 58″	- 10, + cos.	19.9253864
	Zp	= 37. 0. 0.	- - cot.	10.1228856
	pW	= 32. 23. 57	- - - tan.	9.8025008
	Pp	= 23. 28. 0		
	PW	= 55. 51. 57		

Tri. WpZ, WPZ	pW	=32°. 23′. 57″	ar. co. sin.	0.2709855
	PW	=55. 51. 57	sin.	9.9178865
	ZpW	=32. 37. 58	cot.	10.1935941
	AP♈	=67. 29. 8	tan.	10.3824661
	oP♈	=108. 27. 35		
	oPA	=40. 58. 27		
Tri. WPZ	WP	=55. 51. 57	10,+tan.	20.1688210
	AP♈	=67. 29. 8	sin.	9.9655700
	ZP	=57. 56. 36	tan.	10.2032510
Tri. ZPx	ZPx	=40. 58. 27	10, + cos.	19.8779500
	ZP	=57. 56. 36	cot.	9.7967445
	Px	=50. 19. 33	tan.	10.0812055
	Pr	=94. 5. 30		
Tri. ZPx, Zrx	rx	=43. 45. 57	ar. co. sin.	0.1600743
	Px	=50. 19. 33	sin.	9.8863144
	ZPx	=40. 58. 27	tan.	9.9387676
	Zrx	=44. 1. 16	tan.	9.9851563
Tri. Zrx	Zrx	=44. 1. 16	10,+cos.	19.8567795
	rx	=43. 45. 57	tan.	9.9812846
	Zr	=53. 6. 10	cot.	9.8754949

Zr =53. 6. 10 - - - sin. 9.9029362
Hor. par. =61′. 9″=3669″ - log. 3.5645477

rt uncorrected =2934″=48′. 54″ log. 3.4674839

App. zen. dist. Zt =53°.55′.4″ nearly, sin. 9.9075042
Hor. par. =61′. 9″=3669″ - log. 3.5645477

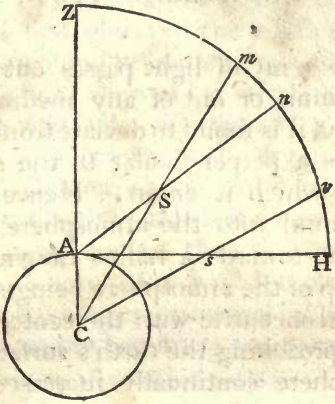
Tri. trs	Par. rt cor. $= 2965'' = 49'.\ 25''$	log.	3.4720519
	$trs = 44°.\ 1'.\ 16''$ - - - -	cos.	9.8567795
	rs par. in $lat. = 2132'' = 35'.\ 32''$	log.	3.3288314
Tri. trs	rt cor. $= 2965''$ - - - - -	log.	3.4720519
	$trs = 44°.\ 1'.\ 16''$ - - - -	sin.	9.8419369
	$ts = 2061'' = 34'.\ 21''$ - - -	log.	3.3139888
	True lat. $ro = 4°.\ 5'.\ 30''$		
	App. lat. $tv = ro + rs = 4°.\ 41'.\ 2''$	cos.	9.9985472
	$ts = 2061''$ - - - -	10, + log.	13.3139888
	ov par. in $long. = 2067'' = 34'.27''$	log.	3.3154416

The value of tv is $ro -$ or $+ rs$, according as the moon has N. or S. latitude.

The order of the signs being from West to East, from A towards C is eastward, and from A towards ♈ is westward; now the parallax depressing the body from r to t, increases the longitude from o to v; but if the point o had been on the other side of A, ov would have lain the contrary way; hence, when the body is to the *East* of the nonagesimal degree, the parallax *increases* the longitude; and when it is to the *West*, it *diminishes* the longitude.

(145.) According to the Tables of *Mayer*, the greatest parallax of the moon (or when she is in her perigee and in opposition) is 61′. 32″; the least parallax (or when in her apogee and conjunction) is 53′. 52″, in the latitude of Paris; the arithmetical mean of these is 57′. 42″; but this is not the parallax at the mean distance, because the parallax varies inversely as the distance, and therefore the parallax at the mean distance is 57′. 24″, an harmonic mean between the two. M. *de Lambre* re-calculated the parallax from the same observations from which *Mayer* calculated it, and found it did not exactly agree with *Mayer's*. He made the equatorial parallax 57′. 11″,4. M. *de la Lande* makes it 57′. 5″ at the equator, 56′. 53″,2 at the pole, and 57′. 1″ for the mean radius of the earth, supposing the difference of the equatorial and polar diameters to be $\frac{1}{300}$ of the whole. From the formula of *Mayer*, the equatorial parallax is 57′. 11″,4.

(146.) To find the mean distance, *Cs*, of the moon, we have *AC*, the mean radius (r) of the earth, : *Cs*, the mean distance (D) of the moon from the earth, :: sin. 57′. 1″ = *AsC* (145) : radius :: 1 : 60,3; consequently $D = 60{,}3\,r$; but $r = 3964$ miles; hence, $D = 239029$ miles.



(147.) According to M. *de la Lande*, the horizontal semidiameter of the moon : it's horizontal parallax for the mean radius (r) of the earth :: 15′ : 54′. 57″,4, or very nearly as 3 : 11; hence, the semidiameter of the moon is $\frac{3}{11}\,r = \frac{3}{11} \times 3964 = 1081$ miles; and as the magnitude of spherical bodies are as the cubes of their radii, we have the magnitudes of the moon and earth as $3^3 : 11^3$, or as 1 : 49 nearly.

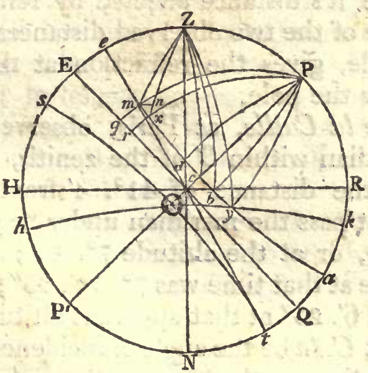
CHAP. VII.

ON REFRACTION.

(148.) WHEN a ray of light passes out of a vacuum into any medium, or out of any medium into one of greater density, it is found to deviate from it's rectilinear course towards a perpendicular to the surface of the medium into which it enters. Hence, light passing out of a vacuum into the atmosphere will, where it enters, be bent towards a radius drawn to the earth's center, the top of the atmosphere being supposed to be spherical and concentric with the center of the earth; and as, in approaching the earth's surface, the density of the atmosphere continually increases, the rays of light, as they descend, are constantly entering into a denser medium, and therefore the course of the rays will continually deviate from a right line, and describe a curve; hence, at the surface of the earth, the rays of light enter the eye of the spectator in a different direction from that in which they would have entered, if there had been no atmosphere; consequently the apparent place of the body from which the light comes, must be different from the true place. Also, the refracted ray must move in a plane perpendicular to the surface of the earth; for, conceiving a ray to come in that plane before it is refracted, then the refraction being always in that plane, the ray must continue to move in that plane. Hence, the refraction is always in a vertical circle. The ancients were not unacquainted with this effect. *Ptolemy* mentions a difference in the rising and setting of the stars in different states of the atmosphere; he makes, however, no allowance for it in his computations from his observations; this correction, therefore, must be applied, where great accuracy is required. *Archimedes* observed the same in water, and thought the quantity of refraction was in proportion to the angle of incidence. *Alhazen*, an Arabian optician, in the eleventh century, by observing the distance of a circumpolar star from the pole, both above and below, found them to be different, and such as ought to arise from refraction. *Snellius*, who first observed the relation between the angles of incidence and refraction, says, that *Waltherus*, in his computation, allowed for refraction; but *Tycho* was the first person who constructed a table for that purpose, which, however, was very incorrect, as he supposed the refraction at 45° to be nothing. About the year 1660, *Cassini* published a new table of refractions, much more correct than that of *Tycho*; and, since his time, astronomers have employed much attention in constructing more correct tables, the niceties of modern astronomy requiring their utmost accuracy.

To find the quantity of refraction.

(149.) *First method.* Take the altitude of the sun, or a star whose right ascension and declination are known, and note the time by the clock; observe also the times of their transits over the meridian, and that interval gives the hour angle. Now, in the triangle PZx, we know PZ and Px, the complements of latitude and declination, and the angle xPZ, to find the



side Zx (92), the complement of which is the *true* altitude, the difference between which and the *observed* altitude, is the refraction at that altitude.

Ex. On May 1, 1738, at 5*h*. 20′ in the morning, *Cassini* observed the altitude of the sun's center at Paris to be 5°. 0′. 14″, and the sun passed the meridian at 12*h*. 0′. 0″, to find the refraction, the latitude being 48°. 50′. 10″, and the declination was 15°. 0′. 25″. The sun's distance from the meridian was 6*h*. 40′, which gives 100° for the hour angle xPZ; also, $PZ = 41°.\ 9'.\ 50''$, and $Px = 74°.\ 59'.\ 35''$; hence, (Trig. Art. 233) $Zx = 85°.\ 10'.\ 8''$, consequently the true altitude was 4°. 49′. 52″. Now to 5°. 0′. 14″, the apparent altitude, add 9″ for the parallax, and we have 5°. 0′. 23″ the apparent altitude corrected for parallax; hence, $5°.\ 0'.\ 23'' - 4°.\ 49'.\ 52'' = 10'.\ 31''$, the refraction at the apparent altitude 5°. 0′. 14″.

(150.) *Second method.* Take the greatest and least altitudes of a circumpolar star which passes through, or very near, the zenith, when it passes the meridian above the pole; then the refraction being nothing in the zenith, we shall have the true distance of the star from the pole at that observation, the altitude of the pole above the horizon, being previously determined; but when the star passes the meridian under the pole, we shall have it's distance affected by refraction, and the difference of the two observed distances, above and below the pole, gives the refraction at the apparent altitude below the pole.

Ex. M. *de la Caille*, at Paris, observed a star to pass the meridian within 6′ of the zenith, and, consequently, at the distance of 41°. 4′ from the pole; hence, it must pass the meridian under the pole at the same distance, or at the altitude 7°. 46′; but the observed altitude at that time was 7°. 52′. 25″; hence, the refraction was 6′. 25″ at that apparent altitude.

(151.) Let CAn be the angle of incidence, CAm the angle of refraction, and consequently mAn the quantity of refraction; let CT be the tangent of Cm, mv

it's sine, nw the sine of Cn, and draw rm parallel to vw; then, as the refraction of air is very small, we may consider mrn as a rectilinear triangle; and hence, by similar triangles, $Av : Am :: rn : mn = \frac{Am \times rn}{Av}$; but Am is constant, and as the ratio of mv to nw is constant by the laws of refraction, their difference, rn, must vary as mv; hence, mn varies as $\frac{mv}{Av}$; but $CT = \frac{Am \times mv}{Av}$, which varies as $\frac{mv}{Av}$, because Am is constant; hence, the refraction, mn, varies as CT, the tangent of the apparent zenith distance of the star, because the angle of refraction, CAm, is the angle between the refracted ray and the perpendicular to the surface of the medium, which perpendicular is directed to the zenith. Whilst, therefore, the refraction is very small, so that rmn may be considered as a rectilinear triangle, this rule will be sufficiently accurate*.

(152.) The twilight in the morning and evening, arises both from the refraction and reflection of the sun's rays by the atmosphere.

It is probable that the reflection arises principally from the exhalations of various kinds with which the lower parts of the atmosphere are charged; for the twilight lasts till the sun is further below the horizon in the evening, than it is in the morning when it begins; and it is longer in summer than in winter. Now, in the *former* case, the heat of the day has raised the vapours and exhalations; and in the *latter*, they will be more elevated from the heat of the season; therefore, upon supposition that the reflection is made by them, the twilight ought to be longer in the evening than in the morning, and longer in summer than in winter.

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* For further information on this subject, see my *Complete System of Astronomy.*

(153.) Another effect of refraction is, that of giving the sun and moon an oval appearance, by the refraction of the lower limb being greater than that of the upper, whereby the vertical diameter is diminished. For suppose the diameter of the sun to be 32′, and the lower limb to touch the horizon, then the mean refraction at that limb is 33″, but the altitude of the upper limb being then 32′, it's refraction is only 28′. 6″, the difference of which is 4′, 54″, the quantity by which the vertical diameter appears shorter than that parallel to the horizon. When the body is not very near the horizon, the refraction diminishing nearly uniformly, the figure of the body is very nearly that of an ellipse.

Chap. VIII.

ON THE SYSTEM OF THE WORLD.

(154.) When any effect or phænomenon is discovered by experiment or observation, it is the business of Philosophy to investigate it's cause. But there are very few, if any, enquiries of this kind, where we can be led from the effect to the cause by a train of mathematical reasoning, so as to pronounce with certainty upon the cause. Sir *I. Newton*, therefore, in his *Principia,* before he treats on the System of the World, has laid down the following Rules to direct us in our researches into the constitution of the universe.

Rule I. No more causes are to be admitted, than what are sufficient to explain the phænomenon.

Rule II. Of effects of the same kind, the same causes are to be assigned, as far as it can be done.

Rule III. Those qualities which are found in all bodies upon which experiments can be made, and which can neither be increased nor diminished, may be looked upon as belonging to all bodies.

Rule IV. In Experimental Philosophy, propositions collected from phænomena by induction, are to be admitted as accurately, or nearly true, until some reason appears to the contrary.

The principles, upon which the application of these Rules is admitted, are, the supposition that the operations of nature are performed in the most simple manner, and regulated by general laws. And although their application may, in many cases, be very unsatisfactory, yet, in the instances to which we shall here want to apply them, their force is little inferior to

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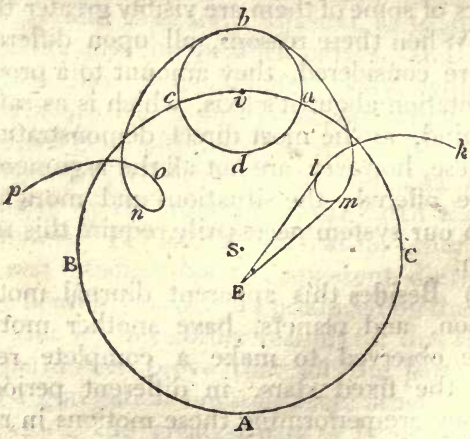
that of direct demonstration, and the mind rests equally satisfied as if the matter were strictly proved.

(155.) The diurnal motion of the heavenly bodies may be accounted for, either by supposing the earth to be at rest, and all the bodies daily to perform their revolutions in circles parallel to each other; or by supposing the earth to revolve about one of it's diameters as an axis, and the bodies themselves to be fixed, in which case their apparent diurnal motions would be the same. If we suppose the earth to be at rest, all the fixed stars must make a complete revolution, in parallel circles, every day. Now the nearest of the fixed stars cannot be less than 400000 times further from us than the sun is, and the sun's distance from the earth is not less than 93 millions of miles. Also, from the discoveries which are every day making by the improvement of telescopes, it appears that the heavens are filled with almost an infinity of stars, to which the number visible to the naked eye bears no proportion; and whose distances are, probably, incomparably greater than what we have stated above. But that an almost infinite number of bodies, most of them invisible, except by the best telescopes, at almost infinite distances from us and from each other, should have their motions so exactly adjusted, as to revolve in the same time, and in parallel circles, and all this without their having any central body, which is a physical impossibility, is an hypothesis, which, by the Rules we have here laid down, is not to be admitted, when we consider, that all the phænomena may be solved, simply by the rotation of the earth about one of it's diameters. If therefore we had no other reason, we might rest satisfied that the apparent diurnal motions of the heavenly bodies are produced by the earth's rotation. But we have other reasons for this supposition. Experiments prove that all the parts of the earth have a gravitation towards each other. Such a body, therefore, the greatest part of whose surface is a fluid, if it remain at rest, must, from the equal gravitation of it's parts, form itself into a perfect sphere. But if we suppose the earth to revolve, the parts most distant from the axis must, from their greater velocity, have a greater tendency to fly off, and therefore that diameter which is perpendicular to the axis must be increased. That this must be the consequence appears from taking an iron hoop, and making it revolve swiftly about one of it's diameters, and that diameter will be diminished, and the diameter perpendicular to it will be increased. Now it appears from mensuration, that the earth is not a perfect sphere, but a spheroid, having the equatorial longer than it's polar diameter. That diameter therefore, about which the earth must revolve, in order to solve all the phænomena of the apparent revolution of the heavenly bodies, is the shortest; and as it necessarily must be the shortest, if the earth be supposed to revolve, this agreement affords a very satisfactory proof of the earth's rotation. Another reason for the earth's rotation is from analogy. The planets are opaque and spherical bodies, like to our earth; now all the planets, on which sufficient observations have been made to determine the matter, are found to revolve about an axis, and the equatorial diameters of some of them are visibly greater than their polar. When these reasons, all upon different principles, are considered, they amount to a proof of the earth's rotation about it's axis, which is as satisfactory to the mind, as the most direct demonstration could be. These, however, are not all the arguments which might be offered; the situations and motions of the bodies in our system necessarily require this motion of the earth.

(156.) Besides this apparent diurnal motion, the sun, moon, and planets, have another motion; for they are observed to make a complete revolution amongst the fixed stars, in different periods; and whilst they are performing these motions in respect to the fixed stars, they do not always appear to move in the same direction, or in that direction in which their complete revolutions are made, but sometimes appear stationary, and sometimes to move in a contrary direction. We will here briefly describe and consider the different systems which have been invented, in order to solve these appearances. *Ptolemy* supposed the earth to be perfectly at rest, and all the other bodies, that is, the sun, moon, planets, comets and fixed stars, to revolve about it every day; but that, besides this diurnal motion, the sun, moon, planets and comets had a motion in respect to the fixed stars, and were situated, in respect to the earth, in the following order; the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn. These revolutions he first supposed to be made in circles about the earth, placed a little out of the center, in order to account for some irregularities of their motions; but as their retrograde motions and stationary appearances could not thus be solved, he supposed them to revolve in epicycloids, in the following manner. Let *ABC* be a circle, *S* the center, *E* the earth, *abcd* another circle, whose center *v* is in the circumference of the circle *ABC*. Conceive the circumference of the circle *ABC* to be carried round the earth every twenty-four hours, according to the order of the letters,



and at the same time let the center *v* of the circle *abcd* have a slow motion in the opposite direction, and let a body revolve in this circle in the direction *abcd*; then it is manifest, that by the motion of the body in this circle, and the motion of the circle itself, the body may describe such a curve as is represented by *klmbnop*; and if we draw the tangents *El, Em,* the body would appear stationary at the points *l* and *m*, and it's motion would be *retrograde* through *lm*, and then *direct* again. Now to make Venus and Mercury always accompany the sun, the centre *v* of the circle *abcd* was supposed to be always very nearly in a right line between the earth and the sun, but more nearly so for Venus than for Mercury, in order to give each it's proper elongation. This system, although it will account for all the motions of the bodies, yet it will not solve the phases of Venus and Mercury; for in this case, in both conjunctions with the sun, they ought to appear dark bodies, and to lose their lights both ways from their greatest elongations; whereas it appears from observation, that in one of their conjunctions they shine with full faces. This system therefore cannot be true.

(157.) The system received by the Egyptians was this: That the earth is immoveable in the center, about which revolve, in order, the Moon, Sun, Mars, Jupiter, and Saturn; and about the Sun revolve Mercury and Venus. This disposition will account for the phases of Mercury and Venus, but not for the apparent motions of Mars, Jupiter, and Saturn.

(158.) The next system which we shall mention, though posterior in time to the true, or *Copernican System*, as it is usually called, is that of *Tycho Brahe*, a Polish nobleman. He was pleased with the Copernican System, as solving all the appearances in the most simple manner; but conceiving, from taking the literal meaning of some passages in scripture, that it was necessary to suppose the earth to be absolutely at rest, he altered the system, but kept as near to it as possible. And he further objected to the earth's motion, because it did not, as he conceived, affect the motions of comets observed in opposition, as it ought; whereas, if he had made observations on some of them, he would have found that their motions could not otherwise have been accounted for. In his system, the earth is supposed to be immoveable in the center of the orbits of the sun and moon, without any rotation about an axis; but he made the sun the center of the orbits of the other planets, which therefore revolved with the sun about the earth. By this system, the different motions and phases of the planets may be solved, the latter of which could not be by the Ptolemaic System; and he was not obliged to retain the epicycloids, in order to account for their retrograde motions and stationary appearances. One obvious objection to this system is, the want of that simplicity by which all the apparent motions may be solved, and the necessity that all the heavenly bodies should revolve about the earth every day; also, it is physically impossible that a large body, as the sun, should revolve in a circle about a small body, as the earth, at rest in it's center; if one body be much larger than another, the center about which they revolve must be very near the large body; an argument which holds also against the Ptolemaic System. It appears also from observation, that the plane in which the sun must, upon this supposition, diurnally move, passes through the earth only twice in a year. It cannot therefore be any force in the earth which can retain the sun in it's orbit; for it would move in a spiral, continually changing it's plane. In short, the complex manner in which all the motions are accounted for, and the physical impossibility of such motions being performed, is a sufficient reason for rejecting this system; especially when we consider, in how simple a manner all these motions may be accounted for, and demonstrated from the common principles of motion. Some of *Tycho's* followers, seeing the absurdity of supposing all the heavenly bodies daily to revolve about the earth, allowed a rotatory motion to the earth, in order to account for their diurnal motion; and this was called the *Semi-Tychonic System*; but the objections to this system are, in other respects, the same.

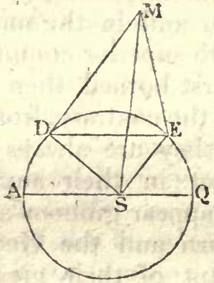
(159.) The system which is now universally received is called the *Copernican*. It was formerly taught by *Pythagoras*, who lived about 500 years before J. C. and *Philolaus*, his disciple, maintained the same; but it was afterwards rejected, till revived by *Copernicus*. Here the Sun is placed in the center of the system, about which the other bodies revolve in the following order; Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgian Planet, which was lately discovered by Dr. Herschel; beyond which, at immense distances, are placed the fixed stars; the moon revolves about the earth, and the earth revolves about an axis. This disposition of the planets solves all the phænomena, and in the most simple manner. For from inferior to superior conjunction, Venus and Mercury appear, first horned, then dichotomized, and next gibbous; and the contrary, from superior to inferior conjunction; they are always retrograde in the inferior, and direct in their superior conjunction. Mars and Jupiter appear gibbous about their quadratures; but in Saturn and the Georgian this is not sensible, on account of their great distances. The motions of the superior planets are observed to be direct in their conjunction, and retrograde in their opposition. All these circumstances are such as ought to take place in the Copernican System. The motions also of the planets are such as should take place upon physical principles. We may also further observe, that the supposition of the earth's motion is necessary, in order to account for a small apparent motion which every fixed star is found to have, and which cannot otherwise be accounted for. The harmony of the whole is as satisfactory a proof of the truth of this system, as the most direct demonstration could be; we shall therefore assume this system.

CHAP. IX.

ON KEPLER'S DISCOVERIES.

(160.) **KEPLER** was the first who discovered the figures of the orbits of the planets to be ellipses, having the sun in one of the foci; this he determined in the following manner.

(161.) Let *S* be the sun, *M* Mars, *D, E,* two places of the earth when Mars is in the same point *M* of it's orbit. When the earth was at *D,* he observed the



difference between the longitudes of the sun and Mars, or the angle *MDS*; in like manner, he observed the angle *MES.* Now the places *D, E,* of the earth in it's orbit being known, the distances *DS, ES,* and the angle *DSE,* will be known; hence, in the triangle *DSE,* we know *DS, SE,* and the angle *DSE,* to find *DE,* and the angles *SDE, SED*; hence, we know the angles *MDE, MED,* and *DE,* to find *MD*; and lastly, in the triangle *MDS,* we know *MD, DS,* and the angle *MDS,* to find *MS,* the distance of Mars from the sun. He also found the angle *MSD,* the difference of the heliocentric longitudes of Mars and the earth. By this method, *Kepler,* from observations made on Mars when in aphelion and perihelion (for he had determined the position of the line of the apsides, by a method which we shall afterwards explain, independent of the form of the orbit), determined the former distance from the sun to be 166780, and the latter 138500, the mean distance of the earth from the sun being 100000; hence, the mean distance of Mars was 152640, and the excentricity of it's orbit 14140. He then determined, in like manner, three other distances, and found them to be 147750, 163100, 166255. He next calculated the same three distances, upon supposition that the orbit was a circle, and found them to be 148539, 163883, 166605; the errors therefore of the circular hypothesis were 789, 783, 350. But he had too good an opinion of *Tycho's* observations (upon which he founded all these calculations) to suppose that these differences might arise from their inaccuracy; and as the distance between the aphelion and perihelion was too great, upon supposition that the orbit was a circle, he knew that the form of the orbit must be an oval; *Itaque planè hoc est: Orbita planetæ non est circulus, sed ingrediens ad latera utraque paulatim, iterumque ad circuli amplitudinem in perigæo exiens, cujusmodi figuram itineris ovalem appellitant*, pag. 213*. And as, of all ovals, the ellipse appeared to be the most simple, he first supposed the orbit to be an ellipse, and placed the sun in one of the foci; and upon calculating the above observed distances, he found they agreed together. He did the same for other points of the orbit, and found that they all agreed; and thus he pronounced the orbit of Mars to be an ellipse, having the sun in one of it's foci. Having determined this for the orbit of Mars, he conjectured the same to be true for all the other planets, and upon trial he found it to be so. Hence, he concluded, *That the six primary planets revolve about the sun in ellipses, having the sun in one of the foci.*

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* See his Work, *De Motibus Stellæ Martis.*

The relative *mean* distances of the planets from the sun are as follows: Mercury, 38710; Venus, 72333; Earth, 100000; Mars, 152369; Jupiter, 520279; Saturn, 954072; Georgian, 1918352.

(162.) Having thus discovered the relative mean distances of the planets from the sun, and knowing their periodic times, he next endeavoured to find if there was any relation between them, having had a strong passion for finding analogies in nature. On March 8, 1618, he began to compare the powers of these quantities, and at that time he took the squares of the periodic times, and compared them with the cubes of the mean distances, but, from some error in the calculation, they did not agree. But on May 15, having made the last calculations again, he discovered his error, and found an exact agreement between them. Thus he discovered that famous law, *That the squares of the periodic times of all the planets are as the cubes of their mean distances from the sun.* Sir *I. Newton* afterwards proved that this is a necessary consequence of the motion of a body in an ellipse, revolving about the focus. *Prin. Phil. Lib.* I. *Sec.* 2. *Pr.* 15.

(163.) *Kepler* also discovered, from observation, that the velocities of the planets, when in their apsides, are inversely as their distances from the sun; whence it followed, that they describe, in these points, equal areas about the sun in equal times. And although he could not prove, from observation, that the same was true in every point of the orbit, yet he had no doubt but that it was so. He therefore applied this principle to find the equation of the orbit (as will be explained in the next Chapter), and finding that his calculations agreed with observations, he concluded that it was true in general, *That the planets describe about the sun, equal areas in equal times.* This discovery was, perhaps, the foundation of the *Principia*, as it might probably suggest to Sir *I. Newton* the idea, that the proposition was true in general, which he afterwards proved it to be. These important discoveries are the foundation of all Astronomy.

(164.) *Kepler* also speaks of *Gravity* as a power which is mutual between all bodies; and tells us, that the earth and moon would move towards each other, and meet at a point as much nearer to the earth than the moon, as the earth is greater than the moon, if their motions did not hinder it. He further adds, that the tides arise from the gravity of the waters towards the moon.

CHAP. X.

ON THE MOTION OF A BOBY IN AN ELLIPSE ABOUT THE FOCUS.

(165.) As the orbits which are described by the primary planets revolving about the sun, are ellipses having the sun in one of the foci, and each describes about the sun equal areas in equal times, we next proceed to deduce, from these principles, such consequences as will be found necessary in our enquiries respecting their motions. From the equal description of areas about the sun in equal times, it appears* that

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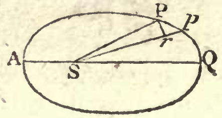
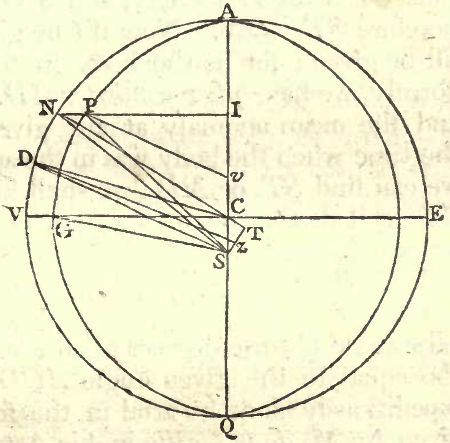
* For if APQ be an ellipse described by a planet about the sun at S in the focus, the indefinitely small area, PSp, described in a



given time, will be constant; draw Pr perpendicular to Sp; and as the area SPp is constant for the same time, Pr varies as $\frac{1}{Sp}$; but the angle pSP varies as $\frac{Pr}{Sp}$, and therefore it varies as $\frac{1}{Sp^2}$; that is, in the *same* orbit, the angular velocity of a planet varies inversely as the square of it's distance from the sun. For *different* planets, the areas described in the same time are not equal, and therefore Pr varies as $\frac{\text{area } SPp}{Sp}$, consequently the angle pSP varies as $\frac{\text{area } SPp}{Sp^2}$; that is, the angular velocities of *different* planets, are as the areas described in the same time directly, and the squares of their distances from the sun inversely.

the planets move with unequal angular velocities about the sun. The proposition therefore, which we here propose to solve, is, given the periodic time of a planet, the time of it's motion from it's aphelion, and the excentricity of it's orbit, to find it's angular distance from the aphelion, or it's *true* anomaly, and it's distance from the sun. This was first proposed by *Kepler*, and therefore goes by the name of *Kepler's Problem.* He knew no direct method of solving it, and therefore did it by very long and tedious tentative operations.

(166.) Let AGQ be the ellipse described by the body about the sun at S in one of it's foci, AQ the major, GC the semi-minor axis, A the aphelion, Q the perihelion, P the place of the body, $AVQE$ a circle, C it's center; draw NPI perpendicular to AQ, join PS, NS, and NC, on which produced let fall the perpendicular ST. Let a body move uniformly in the circle from A to D with the *mean* angular velocity of the body in the ellipse, whilst the body moves in the ellipse from A to P; then the angle ACD is the *mean*, and the angle ASP the *true* anomaly; and the difference of these two angles is called the *equation of*



the planet's center, or *prosthapheresis*. Let p = the periodic time in the ellipse or circle (the periodic times being equal by supposition), and t = the time of describing AP or AD; then, as the bodies in the ellipse and circle describe equal areas in equal times about S and C respectively, we have

area ADC : area of the circle :: $t : p$,
area of the ellipse : area ASP :: $p : t$; also,
area of circ. : area of ellip. :: area ASN : area ASP;*

therefore, area ADC : area ASP :: area ASN : area ASP; hence, $ADC = ASN$; take away the area ACN, which is common to both, and the area $DCN = SNC$; but $DCN = \frac{1}{2} DN \times CN$, and $SNC = \frac{1}{2} ST \times CN$; therefore $ST = DN$. Now if t be given, the arc AD will be given; for as the body in the circle moves uniformly, we have $p : t :: 360° : AD$. Thus we may find the mean anomaly at any given time, knowing the time when the body was in the aphelion; hence, if we can find ST, or ND, we shall know the angle NCA, called the *excentric* anomaly, from whence, by one proportion (167) we shall be able to find the angle ASP the *true* anomaly. The problem is therefore reduced to this; to find a triangle CST, such, that the angle C + the degrees of an arc equal to ST, may be equal to the given angle ACD. This may be expeditiously done by trial in the following manner, given by M. *de la Caille* in his Astronomy. Find what arc of the circumference of the circle $ADQE$ is equal to CA, by saying, 355 : 113 :: 180°. : 57°. 17′. 44″,8, the number of degrees of an arc equal in length to the radius CA; hence, $CA : CS$ :: 57°. 17′. 44″,8 : the degrees of an arc equal to CS. Assume therefore the angle SCT, multiply it's sine into the degrees in CS, and add it to the angle SCT, and if it be equal to the given angle ACD, the supposition was right; if not, add or subtract the difference to or from the first supposition, according as the result is less or greater than ACD, and repeat the operation, and in a very few trials you will get the accurate value of the angle SCT. The degrees in ST may be most readily obtained, by adding the logarithm of CS to the logarithm of the sine of the angle SCT, and subtracting 10 from the index, and the remainder will be the logarithm of the degrees in ST. Having found the value of AN, or the angle ACN, we proceed next to find the angle ASP.

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* See my *Treatise on the Conic Sections, second* edit. prop. 7. of the Ellipse, cor. 3 and 4. And this is the Treatise referred to in the future part of this Work.

(167.) Let v be the other focus, and put $AC = 1$; then by Eucl. B. II. P. 12. $SP^2 - Pv^2 = vS^2 + 2vS \times vI = \overline{vS + 2vI} \times vS = \overline{2Cv + 2vI} \times 2SC = 2CI \times 2SC$; hence, $SP + Pv : 2CI :: 2SC : SP - Pv$, or $2 : 2CI :: 2SC : SP - \overline{2 - SP}$, or $1 : CI :: SC : SP - 1$; hence, $SP = 1 + CS \times CI = 1 + CS \times \text{cos.}\ \angle ACN$. But (Trig. Art. 94), $\dfrac{1 - \text{cos.}\ ASP}{1 + \text{cos.}\ ASP} = \overline{\text{tan.}\ \tfrac{1}{2}\ ASP}^2$; also, (Trig. Art. 125) SP, or $1 + CS \times \text{cos.}\ ACN : \text{rad.} = 1 :: SI$, or $CS + CI$, or $CS + \text{cos.}\ ACN$, : $\text{cos.}\ ASP = \dfrac{CS + \text{cos.}\ ACN}{1 + CS \times \text{cos.}\ ACN}$. Hence, $\overline{\text{tan.}\ \tfrac{1}{2}\ ASP}^2$ $\left(= \dfrac{1 - \text{cos.}\ ASP}{1 + \text{cos.}\ ASP}\right) = \dfrac{1 + CS \times \text{cos.}\ ACN - CS - \text{cos.}\ ACN}{1 + CS \times \text{cos.}\ ACN + CS + \text{cos.}\ ACN}$ $= \dfrac{1 - CS + \text{cos.}\ ACN \times \overline{CS - 1}}{1 + CS + \text{cos.}\ ACN + \overline{CS + 1}} = \dfrac{SQ - \text{cos.}\ ACN \times SQ}{SA + \text{cos.}\ ACN \times SA}$ $= \dfrac{1 - \text{cos.}\ ACN}{1 + \text{cos.}\ ACN} \times \dfrac{SQ}{SA}$ = (Trigonometry, Article 95) $\overline{\text{tan.}\ \tfrac{1}{2}\ ACN}^2 \times \dfrac{SQ}{SA}$; therefore $\sqrt{SA} : \sqrt{SQ} :: \text{tan.}\ \tfrac{1}{2}\ ACN : \text{tan.}\ \tfrac{1}{2}\ ASP$, consequently we get ASP the *true* anomaly.

Ex. Required the true place of *Mercury* on Aug. 26, 1740, at noon, the equation of the center, and it's distance from the sun.

By M. *de la Caille's* Astronomy, Mercury was in it's aphelion on Aug. 9, at 6*h*. 37′. Hence, on Aug. 26, it had passed it's aphelion 16*d*. 17*h*. 23′; therefore 87*d*. 23*h*. 15′. 32″ (the time of one revolution) : 16*d*. 17*h*. 23′ :: 360° : 68°. 26′. 28″, the arc AD, or *mean* anomaly. Now (according to this author) CA : CS :: 1011276 : 211165 (166) :: 57°. 17′. 44″,8 : 11°. 57′. 50″ = 43070″, the value of CS reduced to the arc of a circle, the log. of which is 4,6341749. Also, 68°. 26′. 28″ = 246388″. Assume the angle SCT to be 60° = 216000″, and the operation (166) to find the angle ACN, will stand thus.

4,6341749
9,9375306 log. of . $216000 = a$

4,5717055 37300

253300
246388

$6912 = b$

4,6341749
9,9287987 . . . . . . $209088 = a - b = 58^\circ.\ 4'.\ 48'' = c$

4,5629736 36557

245645
246388

$743 = d$

4,6341749
9,9297694 . . . . . . $209831 = c + d = 58^\circ.17'.11'' = e$

4,5639443 36639

246470
246388

$82 = f$

4,6341749
9,9296626 . . . . . . $209749 = e - f = 58^\circ.15'.49''. = g$

4,5638375 36630

246379
246388

$9 = h$; hence, as the difference between the value deduced from the assumption and the true value, is now diminished about nine times every operation, the next difference would be 1″;

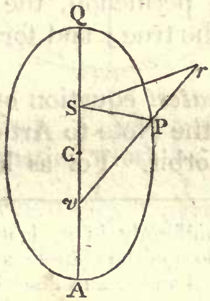
if therefore we add h to g, and then subtract 1″, we get 58°. 15′. 57″, for the true value of the angle ACN, the *excentric* anomaly. Hence, (167) find the *true* anomaly ASP, from the proportion there given, by logarithms, thus.

Log. tang. 29°. 7′. 58″$\frac{1}{2}$ . .	9,7461246
$\frac{1}{2}$ Log. $SQ = 800111$ . . .	2,9515751
	12,6976997
$\frac{1}{2}$ Log. $SA = 1222441$	3,0436141
Log. tang. 24°. 16′. 15″ . .	9,6540856

Hence, the *true* anomaly is 48°. 32′. 30″. Now the aphelion A was in 8^s. 13°. 54′. 30″; therefore the true place of Mercury was 10^s. 2°. 27′. Hence, (166), 68°. 26′. 28″ – 48°. 32′. 30″ = 19°. 53′. 58″, the *equation of the center*. Also, $SP = 1 + CS \times \cos. \angle ACN =$ 1,10983 the distance of Mercury from the sun, the radius of the circle, or the mean distance of the planet, being unity. Thus we are able to compute, at any time, the place of a planet in it's orbit, and it's distance from the sun; and this method of computing the *excentric* anomaly appears to be the most simple and easy of application of all others, and capable of any degree of accuracy.

(168.) As the bodies D and P departed from A at the same time, and will coincide again at Q, ADQ, APQ being described in half the time of a revolution; and as at A the planet moves with it's least angular velocity (by the Note to Art. 165), therefore from A to Q, or in the *first* six signs of anomaly, the angle ACD will be greater than ASP, or the *mean* will be greater than the *true* anomaly; but from Q to A, or in the *last* six signs, as the planet at Q moves with it's greatest angular velocity, the *true* will be greater than the *mean* anomaly. When the equation is greatest in going from A to Q, the *mean* place is before the *true* place, by the equation, and from Q to A, the *true* place is before the *mean* place, by the equation; therefore, from the time the equation is greatest till it becomes greatest again, the difference between the true and mean motions, is twice the equation. From apogee to perigee, the true and mean motions are the same.

(169.) The method ascribed by some writers to *Seth Ward*, Professor of Astronomy at Oxford, and published in 1654, although, as M. *de la Lande* observes, it is given both by *Ward* and *Mercator* to *Bulliardus*, is less accurate than these we have already given; yet as it may, in many cases, serve as a useful approximation, it deserves to be mentioned. He assumed the angular velocity about the other focus v to be uniform*, and therefore made it represent the *mean*



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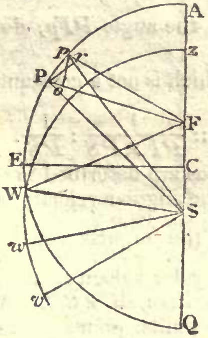
* That this is not true may be thus shown. With the center S and radius $SW=\sqrt{AC\times CE}$ describe the circle zW, then the area of this circle=area of the ellipse (Conic Sections, Ellipse, Prop. 7. Cor. 5); let



anomaly. Produce vP to r, and take $Pr=PS$; then in the triangle Svr, $\overline{rv+vS} : \overline{rv-vS} ::$ tan. $\frac{1}{2}\,.\,\angle\,\overline{vSr+vrS}$: tan. $\frac{1}{2}\,.\,\angle\,\overline{vSr-vrS}$ (Trig. Art. 135); now $\frac{1}{2}\,.\,\overline{rv+vS}=\frac{1}{2}\,AQ+\frac{1}{2}\,vS=AS$, and $\frac{1}{2}\,.\,\overline{rv-vS}=\frac{1}{2}\,AQ-\frac{1}{2}\,vS=SQ$; also, tan. $\frac{1}{2}\,.\,\angle\,\overline{vSr+vrS}$ = tan. $\frac{1}{2}\,\angle\,AvP$, and tan. $\frac{1}{2}\,\angle\,\overline{vSr-vrS}$ = (as $Pr=PS$) tan. $\frac{1}{2}\,.\,\angle\,\overline{vSr-PSr}$ = tan. $\frac{1}{2}\,\angle\,ASP$; hence, *the aphelion distance : perihelion distance :: tan. of $\frac{1}{2}$ the mean anomaly : tan. $\frac{1}{2}$ true anomaly.* This is called the *simple elliptic hypothesis.* In the orbit of the earth, the error is never greater than 17″; in the orbit of the moon, it may be 1′. 35″. By this hypothesis, for 90° from aphelion and perihelion, the computed place is *backwarder* than the true; and for the other part it is *forwarder.*

(170.) The *greatest* equation of the center may be easily found from the Note to Article 169, giving the dimensions of the orbit. For as long as the angular

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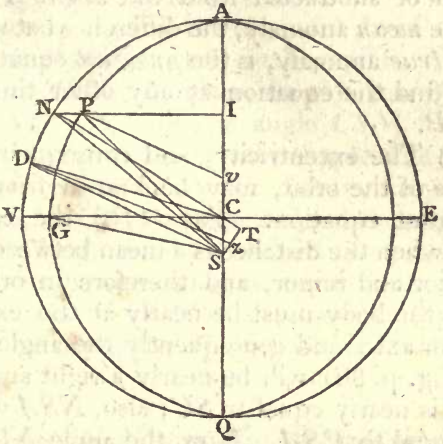
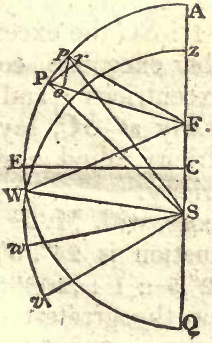
let a body, moving uniformly in it, make one revolution in the same time the body does in the ellipse; and let the bodies set off at the same time from A and z, and describe AP, zv, in the same time; then the angle zSv is the *mean*, and ASP the *true* anomaly. Draw pS indefinitely near to PS, and Pr, po, perpendicular to Sp, FP; then $Pr=po$. Now the angle PFp varies as $\frac{po}{PF}=\frac{Pr}{PF}$; but, in a given time, the area PSp is given, therefore Pr varies as $\frac{1}{PS}$; hence, the angle PFp, described in a given time, varies as $\frac{1}{PF\times PS}$, which is not a constant quantity. Also, $\angle\,PFp : \angle\,PSp :: PS : PF :: \frac{1}{PF\times PS} : \frac{1}{PS^2}$. And by the Note to Art. 165, as equal areas are described in equal times in the circle and ellipse about S, the angular velocity about S in the circle, becomes $\frac{1}{SW^2}$. Hence, the angular velocity about S is greater or less than the mean angular velocity, according as $PF\times PS$ is less or greater than SW^2, or than $AC\times CE$. Also, the angular velocity about S is the same in similar points of the ellipse in respect to the center, or at equal distances from the center.

velocity of the body in the circle is greater than that in the ellipse about *S*, the equation will increase, the



bodies setting out from *A* and *z*; and when they become equal, the equation must be the greatest; this therefore happens when $\frac{1}{SP^2}=\frac{1}{SW^2}=\frac{1}{AC \times CE}$, or when $AC \times CE = SP^2$; hence, *SP* is known. Let *SW* represent this value of *SP*; then as we know



SW, *FW* $(=2AC-SW)$ will be known, and as *SF* is known, we can find the angle *FSW* the *true* anomaly. Hence (Fig. p. 86) (167) $\sqrt{SQ} : \sqrt{SA} ::$ tan. ½ *true* anom. : tan. ½ *excen.* anom. *ACN*, or tan. ½ *SCT*; hence, as we know *SC*, we can find *ST*, or

ND; and to convert that into degrees, say, Rad. = 1 : *ND* :: 57°. 17′. 44″8 : the degrees in *ND*, which added to or subtracted from the angle *ACN*, gives *ACD* the *mean* anomaly, the difference between which and the *true* anomaly, is the *greatest* equation. Thus we may find the equation at any other time, having given *SP*.

(171.) The excentricity, and consequently the dimensions of the orbit, may be known from knowing the greatest equation. For (170) the equation is greatest when the distance is a mean between the semi-axes major and minor, and therefore in orbits nearly circular, the body must be nearly at the extremity of the minor axis, and consequently the angle *NCA*, or *SCT* (Fig. p. 86) will be nearly a right angle, therefore *ST* is nearly equal to *SC*; also, *NSA* will be very nearly equal to *PSA*. Now the angle $NCA - NSA$ $(PSA) = SNC$, and $DCA - NCA = DCN$; add these together, and $DCA - PSA = DCN + SNC$, which (as *NC* is nearly parallel to *DS*) is nearly equal to 2 *DCN*; that is, the difference between the *true* and *mean* anomaly, or the *equation of the center*, is nearly equal to twice the arc *DN*, or twice *ST*, or very nearly twice *SC*. Hence, 57°. 17′. 44″,8 : half the greatest equation :: rad. = 1 : *SC* the excentricity. But if the orbit be considerably excentric, compute the greatest equation to this excentricity; and then, as the equation varies very nearly as *SC*, say, as the computed equation : excentricity found :: given greatest equation : true excentricity.

Ex. If we suppose, with M. *de la Caille*, that Mercury's greatest equation is 24°. 3′. 5″; then 57°. 17′. 44″,8 : 12°. 1′. 32″,5 :: 1 : ,209888 the excentricity very nearly. Now the greatest equation, computed from this excentricity, is 23°. 54′. 28″,5; hence, 23°. 54′. 28″,5 : 24°. 3′. 5″ :: ,209888 : ,211165 the true excentricity. M. *de la Lande* makes the greatest equation 23°. 40′, and the excentricity ,207745.

(172.) The converse of this problem, that is, given the excentricity and true anomaly, to find the mean, may be very readily and directly solved. The excentricity being given, the ratio of the major to the minor axis is known*, which is the ratio of NI to PI; hence, the angle ASP being given, we have $PI : NI :: \tan. ASP : \tan. ASN$; therefore, in the triangle NCS, we know NC, CS, and the angle CSN, to find the angle SCN (Trig. Art. 128), the supplement of which is the angle ACN, or SCT; hence, in the right angled triangle STC, we know SC and the angle SCT, to find ST (Trig. Art. 128), which is equal to ND, the arc measuring the equation, which may be found by saying, radius : ST :: 57°. 17′. 44″,8 : the degrees in ND, which added to ACN, gives ACD the mean anomaly.

To find the hourly Motion of a Planet in it's Orbit, having given the mean *hourly Motion.*

(173.) The hourly motion of a planet in it's orbit is found immediately from the Note to Art. 169; for it appears from thence, that the angles PSp, WSw, described by the body at P in the ellipse, and the body W in the circle in the same time, are as $SW^2 : SP^2$, or as (see Fig. p. 87) $AC \times CE : SP^2$; hence, $PSp = WSw \times \frac{AC \times CE}{SP^2}$ the hourly motion of a planet in it's *orbit*, the angle WSw being the *mean* motion of the planet in an hour. For greater accuracy, SP must be taken at the middle of the hour. Thus we may easily compute a Table of the hourly motions of the planets in their orbits.

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* For as AC, CS are known, we have $GC = \sqrt{GS^2 - SC^2} = \sqrt{AC^2 - SC^2} = \sqrt{\overline{AC + SC} \times \overline{AC - SC}}$, for the computation of which by logarithms, see Trig. Art. 52.

CHAP. XI.

ON THE OPPOSITIONS AND CONJUNCTIONS OF THE PLANETS.

(174.) THE place and time of the opposition of a superior, or conjunction of an inferior planet, are the most important observations for determining the elements of the orbit, because at that time the observed is the same as the true longitude, or that seen from the sun; whereas, if observations be made at any other time, we must reduce the observed to the true longitude, which requires the knowledge of their relative distances, and which, at that time, are supposed not to be known. They also furnish the best means of examining and correcting the tables of the planets' motions, by comparing the computed with the observed places.

(175.) To determine the time of opposition, observe, when the planet comes very near to that situation, the time at which it passes the meridian, and also it's right ascension (111 or 113); take also it's meridian altitude; do the same for the sun, and repeat the observations for several days. From the observed meridian altitudes find the declinations (114), and from the right ascensions and declinations compute (115) the latitudes and longitudes of the planet, and the longitudes of the sun. Then take a day when the difference of their longitudes is nearly 180°, and on that day reduce the sun's longitude, found from observation when it passed the meridian, to the longitude found at the time (t) the planet passed, by finding from observation, or computation, at what rate the longitude then increases. Now in opposition the planet is retrograde, and therefore the difference between the longitudes of the planet and sun increases by the sum of their motions. Hence, the following rule: As the sum (*S*) of their daily motions in longitude: the difference (*D*) between 180° and the difference of their longitudes reduced to the same time (*t*)* (subtracting the sun's longitude from that of the planet to get the difference reckoned from the sun according to the order of the signs) :: 24*h.* : interval between that time (*t*) and the time of opposition. This interval added to or subtracted from the time (*t*), according as the difference of their longitudes at that time was greater or less than 180°, gives the time of opposition. If this be repeated for several days, and the mean of the whole taken, the time will be had more accurately. And if the time of opposition found from observation, be compared with the time by computation from the Tables, the difference will be the error of the Tables, which may serve as means of correcting them.

Ex. On October 24, 1763, M. *de la Lande* observed the difference between the right ascensions of β *Aries*, and *Saturn*, which passed the meridian at 12*h.* 17′. 17″ apparent time, to be 8°. 5′. 7″, the star passing first. Now the apparent right ascension of the star at that time was 25°. 24′. 33″,6; hence, the apparent right ascension of Saturn was 1^{s}. 3°. 29′. 40″,6 at 12*h.* 17′. 17″ apparent time, or 12*h.* 1′. 37″ mean time. On the same day he found, from observation of the meridian altitude of Saturn, that it's declination was 10°. 35′. 20″ N. Hence, from the right ascension and declination of Saturn, it's longitude is found to be 1^{s}. 4°. 50′. 56″, and latitude 2°. 43′. 25″ S. At the same time the sun's longitude was found by calculation to be 7^{s}. 1°. 19′. 22″, which subtracted from 1^{s}. 4°. 50′. 56″, gives 6^{s}. 3°. 31′. 34″; hence, Saturn was 3°. 31′. 34″

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* For this difference shows how far the planet is from opposition; and the proportion is founded on this principal, that the sun approaches the star by spaces in promotion to the times; the spaces *S* and *D* must therefore be as the time 24*h*, and the time (*t*) to opposition.

beyond opposition, but being retrograde will afterwards come into opposition. Now, from the observations made on several days at that time, Saturn's longitude was found to decrease 4′. 50″ in 24 hours, and by computation, the sun's longitude increased 59′. 59″ in the same time, the sum of which is 64′. 49″; hence, 64′. 49″ : 3°. 31′. 34″ :: *24h.* : *78h.* 20′. 20″, which added to October 24, 12*h.* 1′. 37″, gives 27*d.* 18*h.* 21′. 57″ for the time of opposition. Hence, we may find the longitude of Saturn at the time of opposition, by saying, 24*h.* : 78*h.* 20′. 20″ :: 4′. 50″ : 15′. 47″ the retrograde motion of Saturn in 78*h.* 20′. 20″, which subtracted from 1^s. 4°. 50′, 56″, leaves 1^s. 4°. 35′. 9″ the longitude of Saturn at the time of opposition. In like manner we may find the sun's longitude at the same time, in order to prove the opposition; for 24*h.* : 78*h.* 20′. 20″ :: 59′. 59″ : 3°. 15′. 47″, which added to 7^s. 1°. 19′. 22″, the sun's longitude at the time of observation, gives 7^s. 4°. 35′. 9″ for the sun's longitude at the time of opposition, which is exactly opposite to that of Saturn. Hence, we may also find the latitude of Saturn at the same time, by observing, in like manner, the daily variation, or by computation from the Tables after the elements of it's motions are known, and the Tables constructed; by which it appears, that in the interval between the times of observation and opposition, the latitude had increased 6″, and consequently the latitude was 2°. 43′. 31″. Thus we find the times of opposition of all the superior planets.

(176.) The place and time of conjunction of an inferior planet may be found in like manner, when the elongation of the planet from the sun, near the time of conjunction, is sufficient to render it visible; the most favourable time therefore must necessarily be when the geocentric latitude of the planet at the time of conjunction is the greatest. In the year 1689, Venus was in it's inferior conjunction on June 25, and it was observed on 21, 22, and 28; from which observations it's conjunction was found to be at 13*h.* 46′ apparent time at Paris, in longitude ♋ 4°. 53′, 40″, and latitude 3°. 1′. 40″ N. The state of the air must be very favourable, that the time and place of the superior conjunction may be thus observed; for as Venus is then about six times as far from the earth as at it's inferior conjunction, it's apparent diameter and the quantity of light which we receive from it, are so small, as to render it difficult to be perceived. But the most accurate method of observing the time of an inferior conjunction both of Venus and Mercury, is from observations made upon them in their transits over the sun's disc.

CHAP. XII.

ON THE MEAN MOTIONS OF THE PLANETS.

(177.) THE determination of the *mean* motions of the planets, from their conjunctions and oppositions, would very readily follow, if we knew the place of the aphelia and excentricities of their orbits; for then we could (166) find the equation of the orbit, and reduce the *true* to the *mean* place; and the *mean* places being determined at two points of time, give the mean motion corresponding to the interval between the times. But the place of the aphelion is best found from the mean motion. To determine therefore the mean motion, independent of the place of the aphelion, we must seek for such oppositions or conjunctions, as fall very nearly in the same point of the heavens; for then the planet being very nearly in the same point of it's orbit, the equation will be very nearly the same at each observation, and therefore the comparison between the true places will be nearly a comparison of their mean places. If the equation should differ much in the two observations, it must be considered. Now, by comparing the modern observations, we shall be able to get nearly the time of a revolution; and then, by comparing the modern with the ancient observations, the mean motion may be very accurately determined; for any error, by dividing it amongst a great number of revolutions, will become very small in respect to one revolution. As this will be best explained by an example, we shall give one from M. *Cassini* (*Elem. d'Astron.* p. 362), with the proper explanations as we proceed.

Ex. On September 16, 1701, *Saturn* was in opposition at 2*h*. when the place of the sun was ♍ 23°. 21′. 16″, and consequently Saturn in ♓ 23°. 21′. 16″, with 2°. 27′. 45″ south latitude. On September 10, 1730, the opposition was at 12*h*. 27′, and Saturn in ♓ 17°. 53′. 57″, with 2°. 19′. 6″ south latitude. On September 23, 1731, the opposition was at 15*h*. 51′ in ♈ 0°. 30′. 50″, with 2°. 36°. 55″ south latitude. Now the interval of the two first observations was 29 years (of which seven were bissextiles) wanting 5*d*. 13*h*. 33′; and the interval of the two last was 1*y*. 13*d*. 3*h*. 24′. Also, the difference of the places of Saturn in the two first observations was 5°. 27′. 19″, and in the two last it was 12°. 36′. 53″. Hence, in 1*y*. 13*d*. 3*h*. 24′, Saturn had moved over 12°. 36′. 53″; therefore 12°. 36′. 53″ : 5°. 27′. 19″ :: 1*y*. 13*d*. 3*h*. 24′ : 163*d*. 12*h*. 41′, the time of moving over 5°. 27′. 19″ very nearly, because Saturn, being nearly in the same part of it's orbit, will move nearly with the same velocity; this therefore, *added* to the interval between the two first observations (because at the second observation Saturn wanted 5°. 27′. 19″ of being up to the place at the first observation) gives 29 common years 164*d*. 23*h*. 8′, for the time of one revolution. Hence say, 29*y*. 164*d*. 23*h*. 8′ : 365*d*. :: 360°. : 12°. 13′. 23″. 50‴ the *mean* annual motion of Saturn in a common year of 365 days, that is, the motion in a year if it had moved uniformly. If we divide this by 365, we shall get 2′. 0″. 28‴ for the mean daily motion of Saturn. The mean motion thus determined will be sufficiently accurate to determine the number of revolutions which the planet must have made, when we compare the modern with the ancient observations, in order to determine the mean motion more accurately.

The most ancient observation which we have of the opposition of Saturn was on March 2, in the year 228, before J. C. at one o'clock in the afternoon, in the meridian of Paris, Saturn being then in ♍ 8°. 23′, with 2°. 50′ north lat. On February 26, 1714, at 8*h*. 15′, Saturn was found in opposition in ♍ 7°. 56. 46″, with 2°. 3′ north lat. From this time we must subtract 11 days, in order to reduce it to the same style as at the first observation, and consequently this opposition happened on February 15, at 8*h*. 15′. Hence, the difference between these two places was only 26′. 14″. Also, the opposition in 1715 was on March 11, at 16*h*. 55′, Saturn being then in ♍ 21°. 3′. 14″, with 2°. 25′ north lat. Now between the two first oppositions there were 1942 years (of which 485 were bissextiles) wanting 14*d*. 16*h*. 45′, that is, 1943 common years, and 105*d*. 7*h*. 15′ over. Also, the interval between the times of the two last oppositions was 378*d*. 8*h*. 40′, during which time Saturn had moved over 13°. 6′. 28″; hence, 13°. 6′. 28″ : 26′. 14″ :: 378*d*. 8*h*. 40′ : 13*d*. 14*h*. which added to the time of the opposition in 1714, gives the time when the planet had the same longitude as at the opposition in 228 before J. C. This quantity added to 1943 common years 105*d*. 7*h*. 15′, gives 1943*y*. 118*d*. 21*h*. 15′, in which interval of time Saturn must have made a certain complete number of revolutions. Now having found, from the modern observations, that the time of one revolution must be nearly 29 common years 164*d*. 23*h*. 8′, it follows that the number of revolutions in the above interval was 66; dividing therefore that interval by 66, we get 29*y*. 162*d*. 4*h*. 27′ for the time of one revolution. From comparing the oppositions in the years 1714 and 1715, the true movement of Saturn appears to be very nearly equal to the mean movement, which shows that the oppositions have been observed very near the mean distance; consequently the motion of the aphelion cannot have caused any considerable error in the determination of the mean motion. Hence, the mean annual motion is 12°. 13′. 35″. 14‴, and the mean daily motion 2′. 0″. 35‴. Dr. *Halley* makes the annual motion to be 12°. 13′. 21″. M. *de Place* makes it 12°. 13′. 36″,8. As the revolution here determined is that in respect to the *longitude* of the planet, it must be a *tropical* revolution. Hence, to get the sidereal revolution, we must say, 2′. 0″. 35‴ : 24′. 42″. 20‴ (the precession in the time of a tropical revolution, Art. 130) :: 1 day : 12*d*. 7*h*. 1′. 57″, which added to 29*y*. 162*d*. 4*h*. 27′, gives 29*y*. 174*d*. 11*h*. 28′. 57″ the length of a sidereal year of Saturn. Thus we find the periodic times of all the superior planets. The periodic times of the inferior are found from their conjunctions.

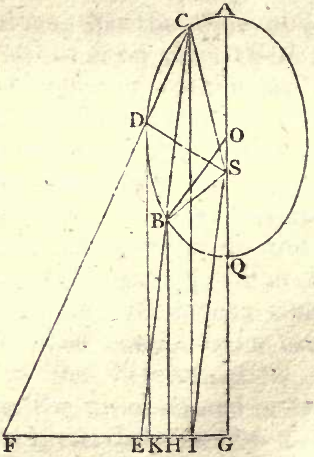
The periodic times of the planets are as follows; Mercury, 87*d*. 23*h*. 15′. 43″,6 ; Venus, 224*d*. 16*h*. 49′. 10″,6 ; Mars, 1*y*. 321*d*. 23*h*. 30′. 35″,6; Jupiter, 11*y*. 315*d*. 14*h*. 27′. 10″,8 ; Saturn, 29*y*. 174*d*. 1*h*, 51′, 11″,2 ; the Georgian, 83*y*. 150*d*. 18*h*.

CHAP. XIII.

ON THE GREATEST EQUATION, EXCENTRICITY, AND PLACE OF THE APHELIA, OF THE ORBITS OF THE PLANETS.

(178.) HAVING determined the mean motions of the planets, we proceed next to show the method of finding the greatest equation of their orbits, the excentricity, and place of their aphelia. For although, in order to determine the mean motions very accurately, these things were supposed to be known, yet, without them, the mean motions may be so nearly ascertained, that these elements may from thence be very accurately settled. By Art. 161, we may find the distance of a planet from the sun in any point of it's orbit. The problem therefore is, having given in length and position, three lines drawn from the focus of an ellipse, to determine the ellipse.

(179.) Let SB, SC, SD, be the three lines; produce CB, CD, and take $SB : SC :: EB : EC$, and



$SC : SD :: CF : DF$, then $SC - SB : SC :: BC : EC = \frac{SC \times BC}{SC - SB}$, and $SC - SD : SC :: DC : CF =$

$\frac{SC \times DC}{SC - SD}$. Join FE, and draw DK, CI, BH, perpendicular to it. Now, by similar triangles, $IC : HB :: EC : EB ::$ (by const.) $SC : SB$; also, $IC : KD :: CF : DF :: SC : SD$. Hence, the proportion of IC, HB, KD, is the same as SC, SB, SD, consequently EF is the directrix of the ellipse passing through B, C, D, (Con. Sect. p. 31). Through S draw $ASQG$ perpendicular to FE; take $GA : AS :: CI : CS$, and $GQ : SQ :: CI : CS$; then $CI + CS : CS :: GS : SQ = \frac{CS \times GS}{CI + CS}$; in like manner we find $AS = \frac{CS \times GS}{CI - CS}$, and A, Q, will be the vertices of the conic section.

(180.) *Calculation.* In the triangles SBC, SCD, we know two sides and the included angles, they being the distances of the observed places in the orbit; hence, (Trig. Art. 135) we can find BC, CD, and the angles BCS, SCD, and consequently BCD. Hence (179) we know CE and CF, and the angle ECF being also known, the angle CEF can be found. Therefore in the right angled triangle CIE, CE and the angle E are given; hence, (Trig. Art. 128) CI is known. Join SI; then in the triangle SIC we know CI, CS and the angle SCI $(= BCS - ECI)$; hence, we know SI, and the angles CIS, CSI, and hence the angle SIG is known; therefore, in the right angled triangle SIG, we know SI and the angle SIG, from whence SG is found. Hence, (179) we know SA, SQ, half the difference of which is the excentricity, and their sum $= AQ$. Lastly, in the triangle BSO (O being the other focus) we know all the sides, to find the angle BSA (Trig. Art. 133), the distance of the aphelion from the observed place B.

In the year 1740, on July 17, August 26, September 6, M. *de la Caille* found three distances of Mercury (the mean distance being 10000) as follows: SB

= 10351,5, *IC* = 11325,5, *SD* = 9672,166, the angle *BSC* = 3^{s}. 27°. 0′. 35″, and *CSD* = 44°. 40′. 4″. Hence, *BCS* = 29°. 55′. 5″, *BC* = 18941, *SCD* = 56°. 49′, *CD* = 8124,5, *BCF* = 86°. 44′. 5″, *CE* = 215004, *CE* = 55647, *CEF* = 14°. 41′. 44″, *CI* = 54543, *CSI* = 124°. 47′. 45″, *CIS* = 9°. 49′. 4″, *SI* = 47281, *SIG* = 80°. 10′. 56″, *SG* = 46589, *SP* = 8010,5, *SA* = 12209, *SO* = 4198,5; hence, the excentricity = 2099,75, *BSA* = 71°. 37′. 23″, or 2^{s}. 11°. 37′. 23″, which added to 6^{s}. 2°. 13′. 51″, the position of *SB*, gives 8^{s}. 13°. 51′. 14″ for the place of the aphelion. Hence, the greatest equation is 24°. 3′. 5″.

(181.) Or from the same data, the place of the aphelion and excentricity may be thus found. Put ths semi-axis major = 1, $SB = a$, $SD = b$, $SC = c$, the angle $BSD = v$. $BSC = u$, $BSQ = x$, $OS = e$, half the parameter $= r$. Then, by a well-known property of the ellipse (Conic Sect. Ellipse, Prop. 16), $a = \frac{r}{1 + e\cos. x}$, $b = \frac{r}{1 + e.\cos.\overline{v+x}}$, $c = \frac{r}{1 + e.\cos.\overline{u+x}}$; hence, $r = a + ae.\cos. x = b + be.\cos.\overline{v+x} = c + ce.\cos.\overline{u+x}$; therefore, $\frac{b-a}{a.\cos.x - b.\cos.\overline{v+x}} = e = \frac{c-a}{a.\cos.x - c.\cos.\overline{u+x}}$; now for $\cos.\overline{v+x}$, and $\cos.\overline{u+x}$, substitute $\cos. v. \cos. x - \sin. v \sin. x$, and $\cos. u. \cos. x - \sin. u. \sin. x$ (Trig. Art. 102), and we shall have

$$\frac{b-a}{a.\cos. x - b.\cos. v.\cos. x + b.\sin. v \sin. x} = \frac{c-a}{a.\cos. x - c.\cos. u.\cos. x + c.\sin. u.\sin. x};$$

divide each denominator by cos. x, and we have

$$\frac{b-a}{a - b.\cos. v + b.\sin. v.\tan. x} = \frac{c-a}{a - c.\cos. u + c \sin. u.\tan. x};$$

hence, $\tan. x = \frac{b.\overline{c-a}.\cos. v - c.\overline{b-a}.\cos. u - a.\overline{c-b}}{b.\overline{c-a}.\sin. v - c.\overline{b-a}.\sin. u}$, which gives the place of the perihelion. Hence, we know $e = \frac{c-a}{a\,.\cos.\,x - c\cos.\,\overline{u+x}}$ the excentricity; consequently $1-e$ and $1+e$, the perihelion and aphelion distances, are known. The *species* of the ellipse being determined, it's major axis may be thus found: Compute the *mean* anomaly corresponding to the angle *CSB*, then say, as that mean anomaly : 360° :: the time of describing the angle *CSB* : the periodic time. The periodic time being known, the major axis is found (162) by *Kepler's* Rule. For other practical methods, see my *Complete System of Astronomy*.

(182.) All the epochs in our Astronomical Tables are reckoned from noon on December 31, in the common years, and from January 1, in the bissextiles.

The places of the aphelia for the beginning of 1750, are, Mercury, 8^{s}. 13°. 33′. 58″; Venus, 10^{s}. 7°. 46′. 42″; the Earth, 3^{s}. 8°. 37′. 16″; Mars, 5^{s}. 1°. 28′. 14″; Jupiter, 6^{s}. 10°. 21′. 4″; Saturn, 8^{s}. 28°. 9′. 7″; the Georgian, 11^{s}. 16°. 19′. 30″.

The excentricities of the orbits, the mean distance of the earth from the sun being 100000, are, Mercury, 7955,4; Venus, 498; the Earth, 1681,395; Mars, 14183,7; Jupiter, 25013,3; Saturn, 53640,42; the Georgian, 90804.

The greatest equations are, Mercury, 23°. 40′. 0″; Venus, 0°. 47′. 20″; the Earth, 1°. 55′. 36″,5; Mars, 10°. 40′. 40″; Jupiter, 5°. 30′. 38″,3; Saturn, 6°. 26′. 42″; the Georgian, 5°. 27′. 16″.

The aphelia of the orbits of the planets have a motion, which may be found, from finding the places of the aphelia of each at two different times. Those motions in longitude in 100 years are, Mercury, 1°. 33′. 45″; Venus, 1°. 21′. 0″; the Earth, 1°. 43′. 35″; Mars, 1°. 51′. 40″; Jupiter, 1°. 34′. 33″; Saturn, 1°. 50′. 7″.

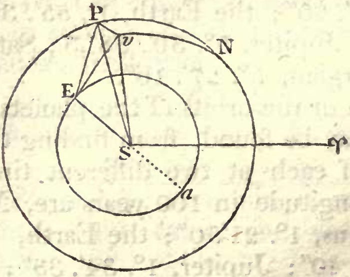
According to the calculation of M. *de la Grange*, the aphelion of the *Georgian* Planet is progressive 3″,17 in the year, from the action of Jupiter and Saturn; consequently it's motion in longitude is 50″,25 + 3″,17 = 53″,42.

CHAP. XIV.

ON THE NODES AND INCLINATIONS OF THE ORBITS OF THE PLANETS TO THE ECLIPTIC.

(183.) FROM observing the course of the planets for one revolution, their orbits are found to be inclined to the ecliptic, for they appear only twice in a revolution to be in the ecliptic; and as it is frequently requisite to reduce their places in the ecliptic, ascertained from observation, to the corresponding places in their orbits, it is necessary to know the inclinations of their orbits to the ecliptic, and the points of the ecliptic where their orbits intersect it, called the *Nodes*. But, previous to this, we must show the method of reducing the places of the planets seen from the earth to the places seen from the sun, and how to compute the heliocentric latitudes.

(184.) Let *E* be the place of the earth, *P* the planet, *S* the sun, ♈ the first point of aries; draw *Pv* perpendicular to the ecliptic, and produce *Es* to *a*. Compute, at the time of observation, the longitude of the sun seen at *a* (115), and you have the longitude of the earth at *E*, or the angle ♈ *SE*; compute also the longitude of the planet, or the angle ♈ *Sv* (115),

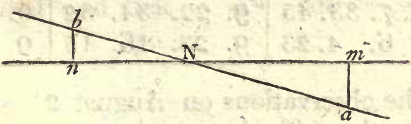


and the difference of these two angles is the angle ESv of *commutation.* Observe the place of the planet in the ecliptic; and the place of the sun being known, we have the angle vES of elongation in respect to longitude; hence, we know the angle SvE, which measures the difference of the places of the planets seen from the earth and the sun; therefore, the place of the planet seen from the earth being known, the place seen from the sun will be known. Also,

tan. PEv : rad. :: vP : Ev (Trig. Art. 123)
rad. : tan. PSv :: vS : vP

∴ tan. PEv : tan. PSv :: vS : Ev :: sin. SEv : sin. ESv; that is, *the sine of elongation in longitude* : *sin. of the difference of the longitudes of the earth and planet* :: *tan. of the geocentric latitude* : *tan. of the heliocentric latitude.* When the latitude is small, Sv : Ev is very nearly as PS : PE, which, in opposition, is very nearly as PS : $PS - SE$. Or we may compute (167) the values of PS and SE, which we can do with more accuracy than we can compute the angles SEv and ESv. The *curtate* distance Sv of the planet from the sun may be found, by saying, rad. : cos. PSv :: PS : Sv.

(185.) Now to determine the place of the node, find the planet's heliocentric latitudes just before and after it has passed the node, and let a and b be the places in the orbit, m and n the places reduced to the ecliptic; then the triangles amN, bnN (which we



may consider as rectilinear) being similar, we have $am : bn :: Nm : Nn$; therefore, $am + bn : am :: Nm + Nn \; (mn) : Nm$, or $am + bn : mn :: am : Nm$, that is, *the sum of the two latitudes* : *the difference of the longitudes* :: *either latitude* : *the distance of the node from the longitude corresponding to that latitude.* Or if we take the two latitudes from the earth, it will be very nearly as accurate when the observations are made in opposition. If the distance of the observations should exceed a degree, this rule will not be sufficiently accurate, in which case we must make our computations for spherical triangles thus. Put $mn = a$, $am = \beta$, $bn = b$, $Nm = x$; then (Trig. Art. 212) $\frac{\sin.\ \overline{a-x}}{\tan.\ b} = \cot.\ N = \frac{\sin.\ x}{\tan.\ \beta}$, radius being unity; but (Trig. Art. 101) $\sin.\ \overline{a-x} = \sin.\ a \times \cos.\ x - \sin.\ x \times \cos.\ a$; hence, $\frac{\sin.\ a \times \cos.\ x - \sin.\ x \times \cos.\ a}{\tan.\ b} = \frac{\sin.\ x}{\tan.\ \beta}$; therefore, $\frac{\sin.\ a \times \tan.\ \beta}{\tan.\ b + \cos.\ a \times \tan.\ \beta} = \frac{\sin.\ x}{\cos.\ x} = \tan.\ x$.

Ex. Mr. *Bugge* observed the right ascension and declination of *Saturn*, and thence deduced (114, 184) the following heliocentric longitudes and latitudes.

1784. *Apparent Time.*	*Heliocentric Long.*	*Heliocentric Lat.*
July 12, at 12^h. 3'. 1''	9^s. 20°. 37'. 29''	0°. 3'. 13'' N.
20, – 11. 29. 9	9. 20. 51. 53	0. 2. 41
Aug. 1, – 10. 38. 25	9. 21. 13. 17	0. 1. 34
8, – 10. 9. 0	9. 21. 26. 2	0. 0. 56
21, – 9. 14. 59	9. 21. 49. 27	0. 0. 2
27, – 8. 50. 19	9. 22. 0. 12	0. 0. 27 S.
31, – 8. 33. 47	9. 22. 7. 32	0. 0. 50
Sept. 5, – 8. 13. 45	9. 22. 16. 28	0. 1. 21
15, – 7. 33. 45	9. 22. 34. 32	0. 1. 59
Oct. 8, = 6. 4. 23	9. 23. 16. 15	0. 3. 35

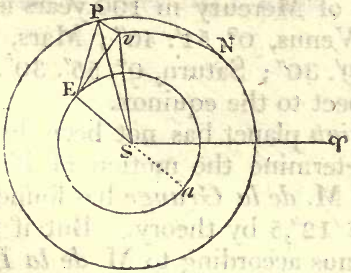
From the observations on August 21 and 27, by considering the triangles as plane, $x = 44'',5$; from those on 21 and 31, $x = 44'',5$; and from those on August 21, and September 5, $x = 40''$; the mean of these is $x = 42''$; Mr. *Bugge* makes $x = 41''$, probably by taking the mean of a greater number, or computing from considering them as spherical triangles; hence, the heliocentric place of the descending node was 9^s.

21°. 50′. 8″,5. Now on August 21, at 9*h*. 12′. 26″ true time, *Saturn*'s heliocentric longitude was 9^s. 21°. 49′. 27″, and on 27, at 8*h*. 49′. 23″ true time, it was 9^s. 22°. 0′. 12″; therefore, in 5*d*. 23*h*. 36′. 57″ Saturn moved 10′. 45″ in longitude; hence, 10′. 45″ : 41″ :: 5*d*. 23*h*. 36′. 57″ : 9*h*. 7′. 44″ the time of describing 41″ in longitude, which added to August 21, 9*h*. 12′. 26″, gives August 21, 18*h*. 20′. 10″, the time when Saturn was in it's node.

The longitudes of the nodes of the planets for the beginning of 1750, are, Mercury, 1^s. 15°. 20′. 43″; Venus, 2^s. 14°. 26′. 18″; Mars, 1^s. 17°. 38′. 38″; Jupiter, 3^s. 7°. 55′. 32″; Saturn, 3^s. 21°. 32′. 22″; Georgian, 2^s. 12°. 47′.

(186.) To determine the inclination of the orbit, we have *am* the latitude of the planet, and *mN* it's distance upon the ecliptic from the node; hence, (Trig. Art. 210) sin. *mN* : tan. *am* :: rad. : tan. of the angle *N*. But the observations which are near the node must not be used to determine the inclination, as a very small error in the latitude will make a considerable error in the angle. If we take the observation on July 20, it gives the angle 2°. 38′. 15″; if we take that on October 8, it gives the angle 2°. 22′. 13″; the mean of these is 2°. 30′. 14″, the inclination of the orbit to the ecliptic, from these observations. Or the inclination may be found thus.

(187.) Find the angle *PSv* (184), then the place of the planet and that of it's node being given, we know *vN*; hence (Trig. Art. 210), sin. *vN* : tan. *Pv* :: rad. : tan. *PNv* the inclination of the orbit.



On March 27, 1694, at 7*h*. 4′. 40″, at Greenwich, Mr. *Flamstead* determined the right ascension of *Mars* to be 115°. 48′. 55″, and it's declination 24°. 10′. 50″ north; hence, (184) the geocentric longitude was ♋ 23°. 26′. 12″, and latitude 2°. 46′. 38″. Let S be the sun, E the Earth, P Mars, v it's place reduced to the ecliptic. Now the true place of Mars (by calculation) seen from the sun was ♌ 28°. 44′. 14″, and the place of the sun was ♈ 7°. 34′. 25″; hence, subtracting the place of the sun from the place of Mars seen from the earth, we have the angle vES between the sun and Mars 105°. 51′. 47″; and the place of the earth being ♎ 7°. 34′. 25″, take from it the place of Mars, and we have the angle ESv = 38°. 50′. 11″; hence, (187) sin. 105°. 51′. 47″ : sin. 38°. 50′. 11″ :: tan. PEv = 2°. 46′. 38″ : tan. PSv = 1°. 48′. 36″. Now the place of the node was in ♉ 17°. 15′, which subtracted from ♌ 28°. 44′. 14″, gives 101°. 29′. 14″ for the distance vN of Mars from it's node; hence, sin. vN = 101°. 29′. 14″ : tan. Pv = 1°. 48′. 36″ :: rad. : tan. PNv = 1°. 50′. 50″, the inclination of the orbit. Mr. *Bugge* makes the inclination to be 1°. 50′. 56″,56, for Mar. 1788. M. *de la Lande* makes it 1°. 51′ for 1780.

The inclination of the orbits of the planets are, Mercury, 7°. 0′. 0″; Venus, 3°. 23′. 35″; Mars, 1°. 51′. 0″; Jupiter, 1°. 18′. 56″; Saturn, 2°. 29′. 50″; Georgian, 46′. 20″.

(188.) The motion of the nodes is found, by comparing their places at two different times; from whence, that of Mercury in 100 years is found to be 1°. 12′. 10″; Venus, 0°. 51′. 40″; Mars, 0°. 46′. 40″; Jupiter, 0°. 59′. 30″; Saturn, 0°. 55′. 30″. This motion is in respect to the equinox.

The *Georgian* planet has not been discovered long enough to determine the motion of it's nodes from observation. M. *de la Grange* has found the annual motion to be 12″,5 by theory. But if we take the density of Venus according to M. *de la Lande*, it will be 20″. 40‴, which he uses in his tables.

Thus we have determined all the elements necessary for computing the place of a planet in it's orbit at any time; but to facilitate the operation, which would be extremely tedious if we had only the elements thus given, astronomers have constructed tables of their motions, by which their places at any time may be very readily computed.

Since the discovery of the *Georgium Sidus*, four other primary planets have been discovered: The first, called *Ceres*, was discovered by *M. Piazzi* at *Palermo*, Jan. 1, 1801; the second, called *Pallas*, was discovered by Dr. *Olbers* at *Bremen*, March 28, 1802; the third, called *Juno*, was discovered by *M. Harding* at *Lilienthal*, September 1, 1804; and the fourth, called *Vesta*, was discovered by Dr. *Olbers*, March 29, 1807. The following Table contains the elements of the orbits of the three first; the orbit of the fourth is not yet computed.

Elements.	*Ceres.*	*Pallas.*	*Juno.*
Epoch of Mean Long. 1805 for Merid. of Seeberg	30°.12′.7″,7	18°. 13′. 1″,4	42. 36. 36
Long. of Aphelion - -	326. 28. 4,4	301. 3. 24,3	233. 11. 40
Long. of ascending Node	81. 0. 41	172. 30. 47	171. 4. 15
Inclination of the Orbit -	10. 37. 36	34. 37. 43	13. 3, 38
Excentricity - - - -	0,0784699	0,2461007	0,254236
Log. of Mean Distance -	0,442004	0,4417647	0,4256078
Mean Diurn. Trop. Mot.	771″,0524	771″,6802	815″,959

Dr. *Herschel* makes the diameter of *Pallas* 147 miles; and that of *Ceres* 161,6 miles.

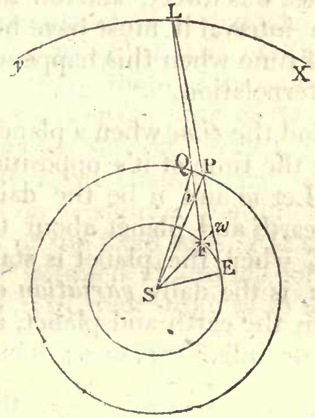
CHAP. XV.

ON THE APPARENT MOTIONS AND PHASES OF THE PLANETS.

(189.) As all the planets describe orbits about the sun as their center, it is manifest, that to a spectator at the sun they would appear to move in the direction in which they really do move, and shine with full faces. But to a spectator on the earth, which is in motion, they will sometimes appear to move in a direction contrary to their real motion, and sometimes appear stationary; and as the same face which is turned towards the earth, is not turned towards the sun, except in conjunction and opposition, some part of the disc which is towards the earth will not be illuminated. These, with some other appearances and circumstances which are observed to take place among the planets, we shall next proceed to explain; and as they are matters in which great accuracy is never requisite, being of no great practical use, but rather subjects of curiosity, we shall consider the motions of all the planets as performed in circles about the sun in the center, and lying in the place of the ecliptic.

(190.) To find the *position* of a planet when stationary. Let *S* be the sun, *E* the earth, *P* the contemporary position of the planet, *Xy* the sphere of the fixed stars to which we refer the motions of the planets; let *EF*, *PQ*, be two indefinitely small arcs described in the same time, and let *EP*, *FQ* produced, meet at *L*; then it is manifest, that whilst the earth moves from *E* to *F*, the planet appears stationary at *L*; and on account of the immense distance of the fixed stars, *EPL*, *FQL* may be considered as parallel. Draw *SE*, *SFw*, *SvP*, and *SQ*; then, as *EP* and

FQ are parallel, the angle $QFS-PES=PwS-PES=ESF$, and $SPw-SQF=SvF-SQF=PSQ$; that is, the cotemporary variations of the angles E and P are as $ESF : PSQ$, the cotemporary variations of the angular velocities of the earth and planet, or (because the angular velocities are inversely as the periodic times, or inversely in the sesquiplicate ratio of the distances) as $SP^{\frac{3}{2}} : SE^{\frac{3}{2}}$, or (if $SP : SE :: a . 1$) as $a^{\frac{3}{2}} : 1^{\frac{3}{2}}$. But sin. SEP : sin. SPE being as $SP : SE$, or, $a : 1$, the cotemporary variations of these angles will be as their tangents*. Hence, if x and y be the sines of the angles SEP and SPE, we have $x : y :: a : 1$, and $\frac{x}{\sqrt{1-x^2}} : \frac{y}{\sqrt{1-y^2}} :: a^{\frac{3}{2}} : 1$, whence $x^2 = \frac{a^3-a^2}{a^3-1} = \frac{a^2}{a^2+a+1}$, and $x = \frac{a}{\sqrt{a^2+a+1}}$ the sine of the planet's elongation from the sun, when stationary.



Ex. If P the earth, and E Venus; and we take the distances of the earth and Venus to be 100000 and 72333, we find $x=0,48264$ the sine of 28°. 51′. 5″,

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[*] See the Optics, Art. 421.

the elongation of Venus when stationary, upon the supposition of circular orbits.

For excentric orbits, the points will depend upon the position of the apsides and places of the bodies at the time. We may, however, get a very near approximation thus. Find the time when the planet would be stationary if the orbits were circular, and compute for several days, about that time, the geocentric place of the planet, so that you get two days, on one of which the planet was direct, and on the other retrograde, in which interval it must have been stationary, and the point of time when this happened may be determined by interpolation.

(191.) To find the *time* when a planet is stationary, we must know the time of it's opposition, or inferior conjunction. Let m and n be the daily angular velocities of the earth and planet about the sun, and v the angle PSE when the planet is stationary; then $m-n$, or $n-m$, is the daily *variation* of the angle at the sun between the earth and planet, according as it is a superior or inferior planet; hence, $m-n$, or $n-m$, : v :: 1 day : $\frac{v}{m-n}$, or $\frac{v}{n-m}$, the time from opposition or conjunction to the stationary points both before and after. Hence, the planet must be stationary twice every *synodic* * revolution.

Ex. Let P be the earth, E Venus; then by the Example to Art. 313, the angle $SPE = 20°. 51',5$; therefore, $PSE = 13°$; also, $n - m = 37'$; hence, $37'$: $13°$. :: 1 day : 21 days the time between the inferior conjunction and stationary positions.

(192.) If the elongation be observed when stationary, we may find the distance of the planet from the sun, compared with the earth's distance, supposed to be unity. For (190) $x^2 = \frac{a^2}{a^2+a+1}$; hence, $a^2 + \frac{x^2}{x^2-1} \times a = -\frac{x^2}{x^2-1} =$ (if $t =$ the tangent of the angle whose sine is x) $a^2 - t^2 a = t^2$; consequently $a = \frac{1}{2} t^2 + t \sqrt{1 + \frac{t^2}{4}}$, upon the supposition of circular orbits.

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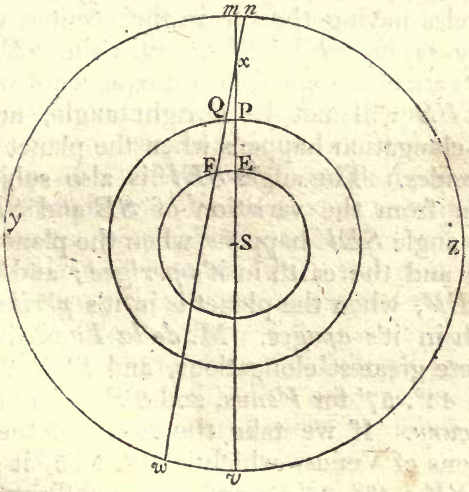
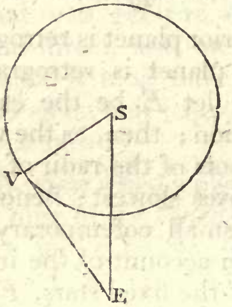
* A Synodic revolution is the time between two conjunctions of the same sort, or two oppositions of a planet.

(193.) A superior planet is retrograde in opposition, and an inferior planet is retrograde in it's inferior conjunction; for let *E* be the earth, *P* a superior planet in opposition; then, as the velocities are as the inverse square roots of the radii of the orbits, the superior planet moves slowest; hence, if *EF*, *PQ*, be two indefinitely small cotemporary arcs, *PQ* is less than *EF*, and on account of the immense distance of the sphere *yZ* of the fixed stars, *FQ* must cut *EP* in some point *x* between *P* and *m*, consequently, the planet appears retrograde from *m* to *n*. If *P* be the earth, and *E* an inferior planet in inferior conjunction, it will appear retrograde from *v* to *w*. These retrograde motions must necessarily continue till the



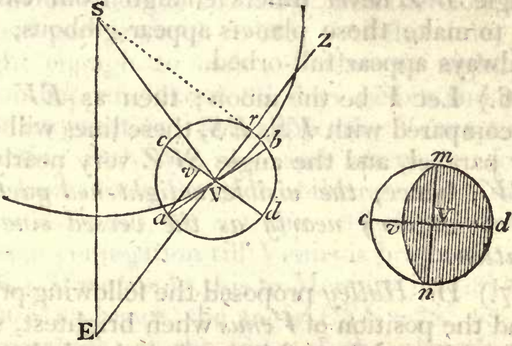
planets become stationary. Hence, from this and the last Article, a superior planet appears retrograde from it's stationary point before opposition to it's stationary point after; and an inferior planet, from it's stationary point before inferior conjunction to it's stationary point after.

(194.) If *S* be the sun, *E* the earth, *V* Venus or Mercury, and *EV* a tangent to the orbit of the planet,



then will the angle *SEV* be the greatest elongation of the planet from the sun; which angle, if the orbits were circles having the sun in their center, would be found by saying, *ES* : *SV* :: rad. : sin. *SEV*. But the orbits are not circular, in consequence of which the angle *EVS* will not be a right angle, unless the greatest elongation happens when the planet is at one of it's apsides. The angle *SEV* is also subject to an alteration from the variation of *SE* and *SV*. The *greatest* angle *SEV* happens, when the planet is in it's *aphelion* and the earth in it's *perigee*; and the *least* angle *SEV*, when the planet is in it's *perihelion* and the earth in it's *apogee*. M. *de la Lande* has calculated these greatest elongations, and finds them 47°. 48′. and 44°. 57′ for *Venus*, and 28°. 20′ and 17°. 36′ for *Mercury*. If we take the mean of the greatest elongations of Venus, which is 46°. 22′5, it gives the angle *VSE* = 43°. 37′,5; and as the difference of the daily *mean* motions of Venus and the earth about the sun is 37′, we have 37′ : 43°. 37′,5 :: 1 day : 70,7 days, the time that would elapse between the greatest elongations and the inferior conjunction, if the motions had been uniform, which will not vary much from the true time.

(195.) To delineate the appearance of a planet at any time. Let S be the sun, E the earth, V Venus,



for example; aVb the plane of illumination perpendicular to SV, cVd, the plane of vision perpendicular to EV, and draw av perpendicular to cd; then ca is the breadth of the visible illuminated part, which is projected by the eye into cv, the versed sine of Cva, or SVZ, for SVc is the complement of each. Now the circle terminating the illuminated part of the planet, being seen obliquely, appears to be an ellipse (Con. Sect. p. 36); therefore, if $cmdn$ represent the projected hemisphere of Venus next to the earth, mn, cd, two diameters perpendicular to each other, and we take $cv =$ the versed sine of SVZ, and describe the ellipse mvn, then cv is the axis minor, and $mcnvm$ will represent the visible enlightened part, as it appears at the earth; and from the property of the ellipse (Con. Sect. Ell. Prop. 7. Cor. 6.), this area varies as cv. Hence, *the visible enlightened part* : *the whole disc* :: *the versed sine of SVZ* : *diameter.*

Hence, *Mercury* and *Venus* will have the same phases from their inferior to their superior conjunction, as the moon has from the new to the full; and the same from the superior to the inferior conjunction, as the moon has from the full to the new. *Mars* will appear gibbous in quadratures, as the angle *SVZ* will then differ considerably from the two right angles, and consequently the versed sine will sensibly differ from the diameter. For *Jupiter, Saturn,* and the *Georgian,* the angle *SVZ* never differs enough from two right angles to make those planets appear gibbous, so that they always appear full-orbed.

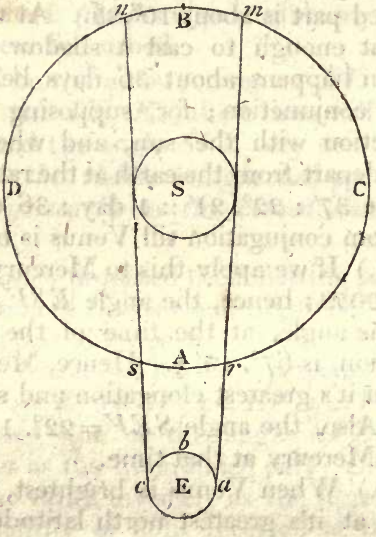
(196.) Let *V* be the moon; then as *EV* is very small compared with *VS, ES,* these lines will be very nearly parallel, and the angle *SVZ* very nearly equal to *SEV*; hence, *the visible enlightened part of the moon varies very nearly as the versed sine of it's elongation.*

(197.) Dr. *Halley* proposed the following problem: To find the position of *Venus* when brightest, supposing it's orbit, and that of the earth, to be circles, having the sun in their center. Draw *Sr* perpendicular to *EVZ*, and put $a=SE$, $b=SV$, $x=EV$, $y=Vr$; then $b-y$ is the versed sine of the angle *SVZ*, which versed sine varies as the illuminated part; and as the intensity of light varies inversely as the square of it's distance, the quantity of light received at the earth varies as $\frac{b-y}{x^2}=\frac{b}{x^2}-\frac{y}{x^2}$; but by Euclid, B. II. P. 12. $a^2=b^2+x^2+2xy$; hence, $y=\frac{a^2-b^2-x^2}{2x}$; substitute this for y, and we get the quantity of light to be as $\frac{b}{x^2}-\frac{a^2-b^2-x^2}{2x^3}=\frac{2bx-a^2+b^2+x^2}{2x^3}$ = a maximum; put the fluxion = 0, and we get $x=\sqrt{3a^2+b^2}-2b$. Now, if $a=1$, $b=,72333$, as in Dr. *Halley's* Tables, then $x=,43036$; hence, the angle $ESV=22°.\ 21'$, but the angle *ESV*, at the time of the planet's greatest elongation, is 43°. 40′; hence, Venus is brightest between it's inferior conjunction and it's greatest elongation; also, the angle $SEV=39°.\ 44'$, the elongation of Venus from the sun at the same time, and $\angle SVZ = VSE + VES = 62^\circ.\, 5'$, the versed sine of which is 0,53, radius being unity; hence (195), the visible enlightened part : whole disc. :: 0,53 : 2; Venus therefore appears a little more than one fourth illuminated, and answers to the appearance of the moon when five days old. Her diameter here is about 39″, and therefore the enlightened part is about 10″,25. At this time, Venus is bright enough to cast a shadow at night. This situation happens about 36 days before and after it's inferior conjunction; for, supposing Venus to be in conjunction with the sun, and when seen from the sun to depart from the earth at the rate of 37′ in 1 day, we have 37′ : 22°. 21′ :: 1 day : 36 days nearly, the time from conjugation till Venus is brightest.

(198.) If we apply this to Mercury, $b = ,3171$, and $x = 1,00058$; hence, the angle $ESV = 78^\circ.\, 55'\frac{2}{3}$; but the same angle, at the time of the planet's greatest elongation, is $67^\circ.\, 13'\frac{1}{2}$. Hence, Mercury is brightest between it's greatest elongation and superior conjunction. Also, the angle $SEV = 22^\circ.\, 18'\frac{3}{4}$, the elongation of Mercury at that time.

(199.) When Venus is brightest, and at the same time is at it's greatest north latitude, it can then be seen with the naked eye at any time of the day, when it is above the horizon; for when it's north latitude is the greatest, it rises highest above the horizon, and therefore is more easily seen, the rays of light having to come through a less part of the atmosphere, the higher the body is. This happens once in about eight years, Venus and the earth returning to the same parts of their orbits after that interval of time.

(200.) *Venus* is a *morning* star from inferior to superior conjunction, and an *evening* star from superior to inferior conjunction. For let *S* be the sun, *E* the earth, *ACBD* the orbit of Venus, *arm*, *csn*, two tangents to the earth, representing the horizon at *a* and *c*. Then the earth revolving about it's axis according to the order *abc*, when a spectator is at *a*, the part *rCm* of the orbit of Venus is above the horizon, but the sun is not yet risen; therefore Venus, in going from *r* through *C* to *m*, appears in the morning before sun-rise. When the spectator is carried by the earth's rotation to *c*, the sun is then set, but the part *nDs* of Venus' orbit is still above the horizon; therefore,



Venus, in going from *n* through *D* to *s*, appears in the evening after sun-set.

(201.) If two planets revolve in circular orbits, to find the time from conjunction to conjunction. Let P = the periodic time of a superior planet, p = that of an inferior, t = the time required. Then $P : 1$ day :: $360° : \frac{360°}{P}$ the angle described by the superior planet in 1 day; for the same reason, $\frac{360°}{p}$ is the angle described by the inferior planet in 1 day; hence, $\frac{360°}{p} - \frac{360°}{P}$ is the daily angular velocity of the inferior planet *from* the superior. Now if they set out from conjunction, they will return into conjunction again after the inferior planet has gained 360°; hence, $\frac{360^\circ}{p} - \frac{360^\circ}{P} : 360^\circ :: 1 \text{ day} : t = \frac{Pp}{P-p}$. This will also give the time between two oppositions, or between any two similar situations.

CHAP. XVI.

ON THE MOON'S MOTION FROM OBSERVATION, AND IT'S PHÆNOMENA.

(202.) THE moon being the nearest, and after the sun, the most remarkable body in our system, and also useful for the division of time, it is no wonder that the ancient astronomers were attentive to discover it's motions; and it is a very fortunate circumstance that their observations have come down to us, as from thence it's mean motion can be more accurately settled, than it could have been by modern observations only; and it moreover gave occasion to Dr. *Halley*, from the observations of some ancient eclipses, to discover an acceleration in it's mean motion. The proper motion of the moon, in it's orbit about the earth, is from west to east; and from comparing it's place with the fixed stars in one revolution, it is found to describe an orbit inclined to the ecliptic; it's motion also appears not to be uniform; and the position of the orbit, and the line of it's apsides are observed to be subject to a continual change. These circumstances, as they are established by observation, we come now to explain.

To determine the Place of the Moon's Nodes.

(203.) The place of the moon's nodes may be determined as in Art. 185, or by the following method.

In a *central* eclipse of the moon, the moon's place at the middle of the eclipse is directly opposite to the sun, and the moon must also then be in the node; calculate therefore the true place of the sun, or, which is more exact, find it's place by observation, and the opposite point will be the true place of the moon, and consequently the place of it's node.

Ex. M. *Cassini*, in his Astronomy, p. 281, informs us, that on April 16, 1707, a central eclipse was observed at Paris, the middle of which was determined to be at 13*h*. 48′ apparent time. Now the true place of the sun, calculated for that time, was 0ˢ. 26°. 19′ 17″; hence, the place of the moon's node was 6ˢ. 26°. 19′. 17″. The moon passed from north to south latitude, and therefore this was the descending node.

(204.) To determine the mean motion of the nodes, find (203) the place of the nodes at different times, and it will give their motion in the interval; and the greater the interval, the more accurately you will get the mean motion. *Mayer* makes the mean annual motion of the nodes to be 12°. 19′. 43″,1.

On the Inclination of the Orbit of the Moon to the Ecliptic.

(205.) To determine the inclination of the orbit, observe the moon's right ascension and declination when it is 90° from it's nodes, and thence compute it's latitude (114), which will be the inclination at that time. Repeat the observation for every distance of the sun from the earth, and for every position of the sun in respect to the moon's nodes, and you will get the inclination at those times. From these observations it appears, that the inclination of the orbit to the ecliptic is variable, and that the *least* inclination is about 5°, which is found to happen when the nodes are in quadratures; and the *greatest* is about 5°. 18′, which is observed to happen when the nodes are in syzygies. The inclination is also found to depend upon the sun's distance from the earth.

On the mean Motion of the Moon.

(206.) The mean motion of the moon is found from observing it's place at two different times, and you get the mean motion in that interval, supposing the moon to have had the same situation in respect to it's apsides at each observation; and if not, if there be a very great interval of the times, it will be sufficiently exact. To determine this, we must compare together the moon's places, first at a small interval of time from each other, in order to get nearly the mean time of a revolution; and then at a greater interval, in order to get it more accurately. The moon's place may be determined directly from observation, or deduced from an eclipse.

(207.) M. *Cassini*, in his Astronomy, p. 294, observes, that on September 9, 1718, the moon was eclipsed, the middle of which eclipse happened at 8*h*. 4′, when the sun's true place was 5ˢ. 16°. 40′. This he compared with another eclipse, the middle of which was observed at 8*h*. 32′. on August 29, 1719, when the sun's place was 5ˢ. 5°. 47′. In this interval of 354*d*. 28′ the moon made 12 revolutions and 349°. 7′ over; divide therefore 354*d*. 28′ by 12 revolutions + 349°. 7′. part of a revolution, and it gives 27*d*. 7*h*. 6′ for the time of one revolution. From two eclipses in 1699, 1717, the time was found to be 27*d*. 7*h*. 43′. 6″.

(208.) The moon was observed at Paris to be eclipsed on Sept. 20, 1717, the middle of which eclipse was at 6*h*. 2′. Now *Ptolemy* mentions, that a total eclipse of the moon was observed at Babylon on March 19, 720 years before J. C. the middle of which happened at 9*h*. 30′, at that place, which gives 6*h*. 48′ at Paris. The interval of these times was 2437 years (of which 609 were bissextiles) 147 days wanting 46′; divide this by 27*d*. 7*h*. 43′. 6″, and it gives 32585 revolutions and a little above $\frac{1}{2}$. Now the difference of the two places of the sun, and consequently of the moon, at the times of observations, was 6ˢ. 6°. 12′. Therefore, in the interval of 2437*y*. 174*d*. wanting 46′, the moon had made 32585 revolutions 6ˢ. 6°. 12′, which gives 27*d*. 7*h*. 43′. 5″ for the mean time of a revolution. This determination is very exact, as the moon was at each time very nearly at the same distance from it's apside. Hence, the mean *diurnal* motion is 13°. 10′. 35″, and the mean *hourly* motion 32′. 56″. 27‴½. M. *de la Lande* makes the mean *diurnal* motion 13°. 10′. 35″,02784394. This is the mean time of a revolution in respect to the equinoxes. The place of the moon at the middle of the eclipse has here been taken the same as that of the sun, which is not accurate, except for a central eclipse; it is sufficiently accurate, however, for this long interval.

(209.) As the precession of the equinoxes is 50″,25 in a year, or about 4″ in a month, the mean revolution of the moon in respect to the fixed stars must be greater than that in respect to the equinox, by the time the moon is describing 4″ with it's mean motion, which is about 7″. Hence, the time of a sidereal revolution of the moon is 27*d.* 7*h.* 43′. 12″.

(210.) Observe accurately the place of the moon for a whole revolution as often as it can be done, and by comparing the true and mean motions, the greatest difference will be double the equation. If two observations be found, where the difference of the true and mean motions is nothing, the moon must then have been in it's apogee and perigee (168). *Mayer* makes the mean excentricity 0,05503568, and the corresponding greatest equation 6°. 18′. 31″,6. It is 6°. 18′. 32″ in his last Tables, published by Mr. *Mason*, under the direction of Dr. *Maskelyne.*

(211.) To determine the place of the apogee, from M. *Cassini*'s observations, we have the greatest equation = 5°. 1′. 44″5; therefore (171), 57°. 17′. 48″,8 : 2°. 30′. 52″,25 :: $AC = 100000$: $CS = 4388$ for the moon's excentricity at that time*. Now (Fig. p. 101.) let v be the focus in which the earth is situated; then

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* The excentricity of the moon's orbit is subject to a variation, it being greatest when the apsides lie in syzygies, and least when in quadratures.

(169) supposing QSP to be the mean anomaly, as QvP is the true anomaly, their difference SPv is the equation of the orbit, which equation is here 37′. 50″,5; and as $PS=Pr$, the angle $vrS=18'.\ 55'',25$; hence, (Trigonometry, Art. 128) $vS=8776 : vr= 200000 :: \sin. vrS=18'.\ 55'',25 : \sin. vSr$, or $QSr,= 7°.\ 12'.\ 20''$, from which take $vrS=18'.\ 55'',25$, and we have $QvP=6°.\ 53'.\ 25''$ the distance of the moon from it's apogee; add this to 2ˢ. 19°. 40′, the true place of the moon, and it gives 2ˢ. 26°. 33′. 25″ for the place of the apogee on December 10, 1685, at 10*h*. 38′. 10″ mean time at Paris. This therefore may be considered as an *epoch* of the place of the apogee.

To determine the mean Motion of the Apogee.

(212.) Find it's place at different times, and compare the difference of the places with the interval of the time between. To do this, we must first compare observations at a small distance from each other, lest we should be deceived in a whole revolution; and then we can compare those at a greater distance. The mean annual motion of the apogee in a year of 365 days is thus found to be 40°. 39′. 50″, according to *Mayer*. *Horrox*, from observing the diameter of the moon, found the apogee subject to an annual equation of 12°,5.

(213.) The motion of the moon having been examined for one month, it was immediately discovered that it was subject to an irregularity, which sometimes amounted to 5° or 6°, but that this irregularity disappeared about every 14 days. And by continuing the observations for different months, it also appeared, that the points where the inequalities were the greatest, were not fixed, but that they moved forwards in the heavens about 3° in a month, so that the motion of the moon, in respect to it's apogee, was about $\frac{1}{120}$ less than it's absolute motion; thus it appeared that the apogee had a progressive motion. *Ptolemy* determined this *first* inequality, or equation of the orbit, from three lunar eclipses observed in the years 719 and 720, before J. C. at Babylon by the Chaldeans; from which he found it amounted to 5°. 1′, when at it's greatest. But he soon discovered that this inequality would not account for all the irregularities of the moon. The distance of the moon from the sun, observed both by *Hipparchus* and himself, sometimes agreed with this inequality, and sometimes it did not. He found that when the apsides of the moon's orbit were in quadratures, this *first* inequality would give the moon's place very well; but that when the apsides were in syzygies, he discovered that there was a further inequality of about $2^\circ\frac{2}{3}$, which made the whole inequality about $7^\circ\frac{2}{3}$. This *second* inequality is called the *Evection*, and arises from a change of excentricity of the moon's orbit. The inequality of the moon was therefore found, by *Ptolemy*, to vary from about 5° to $7^\circ\frac{2}{3}$, and hence the mean quantity was 6°. 20′. *Mayer* makes it 6°. 18′. 31″,6. It is very extraordinary, that *Ptolemy* should have determined this to so great a degree of accuracy. We cannot here enter any further into the inequalities of the motion of the moon. They who wish to see more on this subject, may consult my *Complete System of Astronomy*.

(214.) *Times of the Revolutions of the Moon, of it's Apogee and Nodes, as determined by M.* de la Lande.

Tropical revolution	$27^d.\ 7^h.\ 43'.\ 4'',6795$
Sidereal revolution	27. 7. 43. 11,5259
Synodic revolution	29.12. 44. 2,8283
Anomalistic revolution	27.13. 18. 33,9499
Revolution in respect to the node	27. 5. 5. 35,603
Tropical revolution of the apogee	$8^y.$ 311. 8. 34. 57,6177
Sidereal revolution of the apogee	8. 312.11. 11. 39,4089
Tropical revolution of the node	18. 228. 4. 52. 52,0296
Sidereal revolution of the node	18. 223. 7. 13. 17,744
Diurnal motion of the moon in respect to the equinox	13°. 10'. 35"02784394
Diurnal motion of the apogee	0. 6. 41,069815195
Diurnal motion of the node	0. 3. 10,638603696

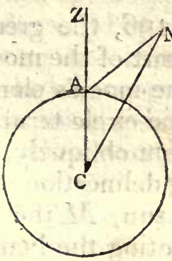
The years here taken are the common years of 365 days.

On the Diameter of the Moon.

(215.) The diameter of the moon may be measured, at the time of it's full, by a micrometer; or it may be measured by the time of it's passing over the vertical wire of a transit telescope; but this must be when the moon passes within an hour or two of the time of the full, before the visible disc is sensibly changed from a circle. To find the diameter by the time of it's passage over the meridian, let d'' = the horizontal diameter of the moon, c = sec. of it's declination, and m = the length of a lunar day, or the time from the passage of the moon over the meridian on the day we calculate, to the passage over the meridian the next day. Then (102) cd'' is the moon's diameter in right ascension; hence, $360° : cd'' :: m :$ the time (t) of passing the meridian; therefore $d'' = 360° \times \frac{t}{cm}$. If we observe when the limb of the moon comes to the meridian, we can find the time when the center comes to it, by adding to, or subtracting from, the time when the first or second limb comes to the meridian, half the time of the passage of the moon over the meridian. The time in which the semidiameter of the moon passes the meridian, may be found by two Tables, in the Tables of the moon's motion.

(216.) *Albategnius* made the diameter of the moon to vary from 29′. 30″ to 35′. 20″, and hence the mean is 32′. 25″. *Copernicus* found it from 27′. 34″ to 35′. 38″, and therefore the mean 31′. 36″. *Kepler* made the mean diameter 31′. 22″. M. *de la Hire* made it 31′. 30″. M. *Cassini* made the diameter from 29′. 30″ to 33′. 38″. M. *de la Lande*, from his own observations, found the mean diameter to be 31′. 26″; the extremes from 29′. 22″ when the moon is in apogee and conjunction, and 33′. 31″ when in perigee and opposition. The mean diameter here taken, is the arithmetic mean between the greatest and least diameters; the diameter at the mean distance is 31′. 7″.

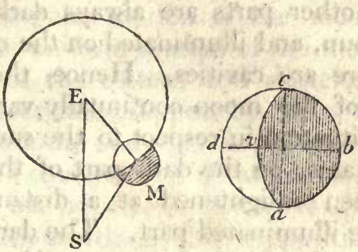
(217.) When the moon is at different altitudes



above the horizon, it is at different distances from the spectator, and therefore there is a change of the apparent diameter. Let C be the center of the earth, A the place of a spectator on it's surface, Z his zenith, M the moon; then (Trig. Art. 128) sin. CAM, or ZAM : sin. ZCM :: CM : $AM = \dfrac{CM \times \text{sin. } ZCM}{\text{sin. } ZAM}$; but the apparent diameter is inversely as it's distance; hence, the apparent diameter varies as $\dfrac{\text{sin. } ZAM}{\text{sin. } ZCM}$, CM being supposed constant. Now, in the horizon, $\dfrac{\text{sin. } ZAM}{\text{sin. } ZCM}$ may be considered as equal to unity; hence, 1 : $\dfrac{\text{sin. } ZAM}{\text{sin. } ZCM}$, or sin. ZCM : sin. ZAM, or cos. true alt. (a) : cos. apparent alt. (A) :: the horizontal diameter : the diameter at the apparent altitude (A). Hence, the horizontal diameter : it's increase :: cos. a : cos. A $-$ cos. a = (Trig. Art. 111) $2 \text{ sin. } \overline{\tfrac{1}{2}a + \tfrac{1}{2}A} \times \text{sin. } \overline{\tfrac{1}{2}a - \tfrac{1}{2}A}$; therefore the increase of the semidiameter $= \text{hor. semidiameter} \times \dfrac{\text{sin. } \overline{\tfrac{1}{2}a + \tfrac{1}{2}A} \times \text{sin. } \overline{\tfrac{1}{2}a - \tfrac{1}{2}A}}{\text{cos. } a}$; from this we may easily construct a table of the increase of the semidiameter for any horizontal semidiameter; and then for any other horizontal semidiameter, the increase will vary in the same proportion.

On the Phases of the Moon.

(218.) By Art. 196, the greatest breadth of the visible illuminated part of the moon's surface, varies as the versed sine of the moon's elongation from the sun, very nearly; and the circle terminating the light and dark part, being seen obliquely, appears an ellipse; hence, the following delineation of the phases. Let E be the earth, S the sun, M the moon; describe the circle $abcd$, representing the hemisphere of the moon which is towards the earth, projected upon the plane of vision; *ac*, *db*, two diameters perpendicular to each other; take dv = the versed sine of elongation SEM,



and describe the ellipse *avc*, and (195) *adcva* will represent the visible enlightened part; which will be horned between conjunction and quadratures; a semicircle at quadratures; and gibbous between quadratures and opposition; the versed sine being less than radius in the first case, equal to it in the second, and greater in the third. The visible enlightened part varying as *dv*, we have, the *visible enlightened part* : *whole* :: *versed sine of elongation* : *diameter*.

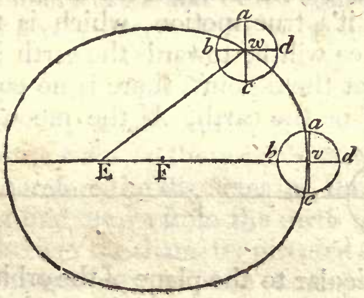
On the Libration of the Moon.

(219.) Many Astronomers have given maps of the face of the moon; but the most celebrated are those of *Hevelius* in his *Selenographia*, in which he has represented the appearance of the moon in it's different states from the new to the full, and from the full to the new; these figures *Mayer* prefers. *Langrenus* and *Ricciolus* denoted the spots upon the surface by the names of Philosophers, Mathematicians, and other celebrated men, giving the names of the most celebrated characters to the largest spots; *Hevelius* marked them with the geographical names of places upon the earth. The former distinction is now generally followed.

The spots upon the moon are caused by the mountains and vallies upon it's surface; for certain parts are found to project shadows opposite to the sun; and when the sun becomes vertical to any of them, they are observed to have no shadows; these therefore are mountains; other parts are always dark on that side next to the sun, and illuminated on the opposite side; these therefore are cavities. Hence, the appearance of the face of the moon continually varies, from it's altering it's situation in respect to the sun. The tops of the mountains, on the dark part of the moon, are frequently seen enlightened at a distance from the cosines of the illuminated part. The dark parts have, by some, been thought to be seas, and by others, to be only a great number of caverns and pits, the dark sides of which, next to the sun, would cause those places to appear darker than others. The great irregularity of the line bounding the light and dark part, on every part of the surface, proves that there can be no very large tracts of water, as such a regular surface would necessarily produce a line, terminating the bright part, perfectly free from all irregularity. If there was much water upon it's surface, and an atmosphere, as conjectured by some Astronomers, the clouds and vapours might easily be discovered by the telescopes which we have now in use; but no such phænomena have ever been observed.

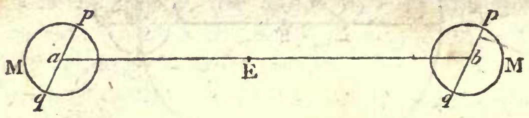
(220.) Very nearly the same face of the moon is always turned towards the earth, it being subject only to a small change within certain limits, those spots which lie near to the edge appearing and disappearing by turns; this is called it's *Libration,* and arises from four causes. 1. *Galileo,* who first observed the spots of the moon after the invention of telescopes, discovered this circumstance; he perceived a small daily variation arising from the motion of the spectator about the center of the earth, which, from the rising to the setting of the moon, would cause a little of the western limb of the moon to disappear, and bring into view a little of the eastern limb. 2. He observed likewise, that the north and south poles of the moon appeared and disappeared by turns; this arises from the axis of the moon not being perpendicular to the plane of it's orbit, and is called a libration in *latitude*. 3. From the unequal angular motion of the moon about the earth, and the uniform motion of the moon about it's axis, a little of the eastern and western parts must gradually appear and disappear by turns, the period of which is a month, and this is called a libration in *longitude*; the cause of this libration was first assigned by *Ricciolus*, but he afterwards gave it up, as he made many observations which this supposition would not satisfy. *Hevelius*, however, found that it would solve all the phænomena of this libration. 4. Another cause of libration arises from the attraction of the earth upon the moon, in consequence of it's spheroidical figure.

(221.) If the angular velocity of the moon about it's axis were equal to it's angular motion about the earth, the libration in *longitude* would not take place. For if *E* be the earth, *abcd* the moon at *v* and *w*, and *avc* be perpendicular to *Ebvd*; then *abc* is that hemisphere of the moon at *v* next to the earth. When the moon comes to *w*, if it did not revolve about it's axis, *bwd* would be parallel to *bvd*, and the same face would not lie towards the earth. But if the moon, by revolving about it's axis in the direction *abcd*, had brought *b* into the line *Ew*, the same face would have been turned towards the earth; and the moon would have revolved about it's axis through the angle *bwE*,



which is equal to the alternate angle *wEe*, the angle which the moon has described about the earth.

(222.) When the moon returns to the same point of it's orbit, the same face is observed to lie towards the earth, and therefore (221) the time of the revolution in it's orbit is equal to the time about it's axis. But in the intermediate points it varies, sometimes a little more to the east, and sometimes to the west, becomes visible; and this arises from it's unequal angular motion *wEv* about the earth, whilst the angular motion about it's axis is equal, in consequence of which these two angles cannot continue equal, and therefore, by the last article, the same face cannot continue towards the earth. Hence, the greatest libration in *longitude* is nearly equal to the equation of the orbit, or about 7° ½ at it's maximum, and would be accurately so, if the axis of the moon were perpendicular to it's orbit; for the difference of the moon's mean motion and true motion, or the equation of the orbit, is the same as the difference of the moon's motion about it's axis and it's true motion, which is the libration. The same face will be towards the earth in apogee and perigee, for at those points there is no equation of the orbit. If *E* be the earth, *M* the moon, *pq* it's axis,



not perpendicular to the plane of the orbit *ab*; then at *a* the pole *p* will be visible to the earth, and at *b* the pole *q* will be visible; as the moon therefore revolves about the earth, the poles must appear and disappear by turns, causing the libration in *latitude*. This is exactly similar to the cause of the variety of our seasons, from the earth's axis not being perpendicular to the plane of it's orbit. Hence, nearly one half of the moon is never visible at the earth. Also, the time of it's rotation about it's axis being a month, the length of the lunar days and nights will be about a fortnight each, they being subject but to a very small change, on account of the axis of the moon being nearly perpendicular to the ecliptic.

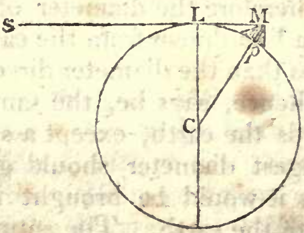
(223.) *Hevelius* (*Selenographia*, p. 245.) observed, that when the moon was at it's greatest north latitude, the libration in latitude was the greatest, the spots which are situated near the northern limb being then nearest to it; and as the moon departed from thence, the spots receded from that limb, and when the moon came to it's greatest south latitude, the spots situated near the southern limb were then nearest to it. This variation he found to be about 1′. 45″, the diameter of the moon being 30′. Hence it follows, then when the moon is at it's greatest latitude, a plane drawn through the earth and moon perpendicular to the plane of the moon's orbit, passes through the axis of the moon; consequently the equator of the moon must intersect the ecliptic in a line parallel to the line of the nodes of the moon's orbit, and therefore, in the heavens, the nodes of the moon's orbit and of it's equator coincide.

(224.) It is a very extraordinary circumstance, that the time of the moon's revolution about it's axis should be equal to that in it's orbit. Sir *I. Newton*, from the altitude of the tides on the earth, has computed that the altitude of the tides on the moon's surface must be 93 feet, and therefore the diameter of the moon perpendicular to a line drawn from the earth to the moon, ought to be less than the diameter directed to the earth, by 186 feet; hence, says he, the same face must always be towards the earth, except a small oscillation; for if the longest diameter should get a little out of that direction, it would be brought into it again by the attraction of the earth. The supposition of D. *de Mairan* is, that that hemisphere of the moon next the earth is more dense than the opposite one, and hence, the same face would be kept towards the earth, upon the same principle as above.

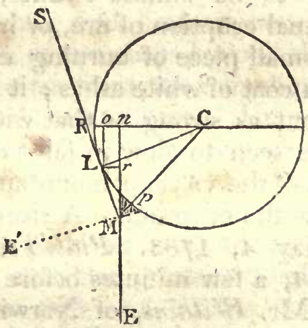
(225.) When the moon is about three days from the new, the dark part is very visible, by the light reflected from the earth, which is moon-light to the Lunarians, considering our earth as a moon to them; and in the most favourable state, some of the principal spots may then be seen. But when the moon gets into quadratures, it's great light prevents the dark part from being visible. According to Dr. *Smith*, the strength of moon-light, at the full moon, is ninety thousand times less than the light of the sun; but, from some experiments of M. *Bouguer*, he concluded it to be three hundred thousand times less. The light of the moon, condensed by the best mirrors, produces no sensible effect upon the thermometer. Our earth, in the course of a month, shows the same phases to the Lunarians, as the moon does to us; the earth is at the full at the time of the new moon, and at the new at the time of the full moon. The surface of the earth being about 13 times greater than that of the moon, it affords 13 times more light to the moon than the moon does to the earth.

On the Altitude of the Lunar Mountains.

(226.) The method used by *Hevelius*, and others since his time, to determine the height of a lunar mountain is this. Let *SLM* be a ray of light from



the sun, passing the moon at *L*, and touching the top of the mountain at *M*; then the space between *L* and *M* appears dark. With a micrometer, measure *LM*, and compare it with *LC*; then, knowing *LC*, we know *LM*, and by Eucl. B. I. p. 47. $CM = \sqrt{CL^2 + LM^2}$ is known; from which subtract *Cp*, and we get the height pM of the mountain. But as Dr. *Herschel* observes, in the *Phil. Trans.* 1781, this method is only applicable when the moon is in quadratures; he has therefore given the following general method. Let E be the earth; draw EMn, and Lo perpendicular to the moon's radius RC, and Lr parallel to on, also ME' perpendicular to SM. Now ML would measure it's full length when seen from the earth in quadratures at E', but seen from E, it only measures the length of Lr. As the plane passing through SM, EM, is perpendicular to a line joining the cusps, the circle RLp may be conceived to be a section of the moon perpendicular to that line. Now it is manifest, that the angle SLo or LCR, is very nearly equal to the elongation of the moon from the sun; and the triangles LrM, LCo, being similar, $Lo : LC :: Lr : LM = \dfrac{LC \times Lr}{Lo} = \dfrac{Lr}{\text{sine of elongation}}$, radius being unity. Hence, we find Mp as before.



Ex. On June, 1780, at seven o'clock, Dr. *Herschel* found the angle under which LM, or Lr appeared, to be 40″,625, for a mountain in the south-east quadrant; and the sun's distance from the moon was 125°. 8′, whose sine is ,8104; hence, 40″,625 divided by ,8104, gives 50″,13, the angle under which LM would appear, if seen directly. Now the semidiameter of the moon was 16′. 2″,6, and taking it's length to be 1090 miles, we have $16'.2'',6 : 50'',13 :: 1090 : LM = 56,73$ miles; hence, $Mp = 1,47$ miles.

(227.) Dr. *Herschel* found the height of a great many more mountains, and thinks he has good reason to believe, that their altitudes are greatly over-rated; and that, a few excepted, they generally do not exceed half a mile. He observes, that it should be examined whether the mountain stands upon level ground, which is necessary, that the measurement may be exact. A low tract of ground between the mountain and the sun will give it higher, and elevated places between will make it lower, than it's true height above the common surface of the moon.

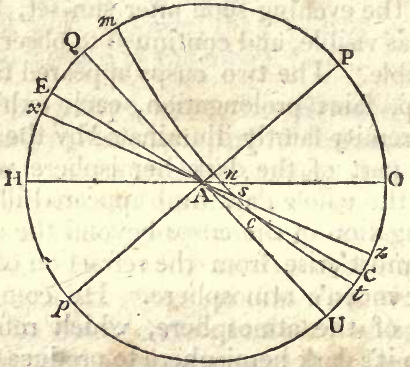
(228.) On April 19, 1787, Dr. *Herschel* discovered three volcanos in the dark part of the moon; two of them seemed to be almost extinct, but the third showed an actual eruption of fire, or luminous matter, resembling a small piece of burning charcoal covered by a very thin coat of white ashes; it had a degree of brightness about as strong as that with which such a coal would be seen to glow in faint day-light. The adjacent parts of the volcanic mountain seemed faintly illuminated by the eruption. A similar eruption appeared on May 4, 1783. *Phil. Trans.* 1787. On March 7, 1794, a few minutes before eight o'clock in the evening, Mr. *Wilkins,* of Norwich, an eminent architect, observed, with the naked eye, a very bright spot upon the dark part of the moon; it was there when he first looked at the moon; the whole time he saw it, it was a fixed, steady light, except the moment before it disappeared, when it's brightness increased; he conjectures that he saw it about five minutes. The same phænomenon was observed by Mr. *T. Stretton,* in St. John's-square, Clerkenwell, London. *Phil. Trans.* 1794. On April 13, 1793, and on February 5, 1794, Mr. *Piazzi,* Astronomer Royal at Palermo, observed a bright spot on the dark part of the moon, near Aristarchus. Several other Astronomers have observed the same phænomenon. See the *Memoirs de Berlin*, for 1788.

(229.) It has been a doubt amongst Astronomers, whether the moon has any atmosphere; some suspecting that at an occultation of a fixed star by the moon, the star did not vanish instantly, but lost it's light gradually; whilst others could never observe any such appearance. M. *Schroetar* of Lilianthan, in the duchy of Bremen, has endeavoured to establish the existence of an atmosphere, from the following observations. 1. He observed the moon when two days and an half old, in the evening soon after sun-set, before the dark part was visible, and continued to observe it till it became visible. The two cusps appeared tapering in a very sharp, faint prolongation, each exhibiting it's farthest extremity faintly illuminated by the solar rays, before any part of the dark hemisphere was visible. Soon after, the whole dark limb appeared illuminated. This prolongation of the cusps beyond the semicircle, he thinks, must arise from the refraction of the sun's rays by the moon's atmosphere. He computes also the height of the atmosphere, which refracts light enough into it's dark hemisphere to produce a twilight, more luminous than the light reflected from the earth when the moon is about 32° from the new, to be 1356 Paris feet; and that the greatest height capable of refracting the solar rays is 5376 feet. 2. At an occultation of Jupiter's satellites, the third disappeared, after having been about 1″ or 2″ of time indistinct; the fourth became indiscernible near the limb; this was not observed of the other two. *Phil. Trans.* 1792. If there be no atmosphere of the moon, the heavens, to a Lunarian, must always appear dark like night, and the stars be constantly visible; for it is owing to the reflection and refraction of the sun's light by the atmosphere, that the heavens, in every part, appear bright in the day.

On the Phænomenon of the Harvest Moon.

(230.) The full moon which happens at, or nearest to, the autumnal equinox, is called the *Harvest* moon; and at that time there is a less difference between the times of it's rising on two successive nights, than at any other full moon in the year; and what we here propose, is to account for this phænomenon.

(231.) Let *P* be the north pole of the equator *QAU*, *HAO* the horizon, *EAC* the ecliptic, *A* the first point of Aries; then, in *north* latitudes, *A* is the ascending node of the ecliptic upon the equator, *AC* being the order of the signs, and *AQ* that of the apparent diurnal motion of the heavenly bodies. When Aries rises in north latitudes, the ecliptic makes the least angle with the horizon; and as the moon's orbit makes but a small angle with the ecliptic, let us first suppose *EAC* to represent the moon's orbit. Let *A* be the place of the moon at it's rising on one night; now, in mean solar time, the earth makes one revolution in $23h.\ 56'.4''$, and brings the same point *A* of the equator to the horizon again; but, in that time, let the moon have moved in it's orbit from *A* to *c*, and draw the parallel of declination *tcm*; then it is manifest that $3'.\ 56''$ before the *same* hour the next night, the moon, in it's diurnal motion, has to describe *cn*



before it rises. Now *cn* is manifestly the least possible, when the angle *CAn* is the least, *Ac* being given. Hence, it rises more nearly at the same hour, when it's orbit makes the least angle with the horizon. Now at the autumnal equinox, when the sun is in the first point of Libra, the moon, at that time at it's full, will be at the first point of Aries, and therefore it rises with the least difference of times, on two successive nights; and it being at the time of it's full, it is more taken notice of; for the same thing happens every month when the moon comes to Aries.

(232.) Hitherto we have supposed the ecliptic to represent the moon's orbit, but as the orbit is inclined to it at an angle of 5°. 9′ at a mean, let *xAsz* represent the moon's orbit when the ascending node is at *A*, and *As* the arc described in a day; then the moon's orbit making the least possible angle with the horizon in that position of the nodes, the arc *sn*, and consequently the difference of the times of rising, will be the least possible. As the moon's nodes make a revolution in about 19 years, the least possible difference can only happen once in that time. In the latitude of London the least difference is about 17′.

(233.) The ecliptic makes the greatest angle with the horizon when the first point of Libra rises, consequently, when the moon is in that part of it's orbit, the difference of the times of it's rising will be the greatest; and if the descending node of it's orbit be there at the same time, it will make the difference the greatest possible; and this difference is about 1*h*. 17′ in the latitude of London. This is the case with the vernal full moons. Those signs which make the least angle with the horizon when they rise, make the greatest angle when they set, and vice versâ; hence, when the difference of the times of rising is the least, the difference of the times of setting is the greatest, and the contrary.

(234.) By increasing the latitude, the angle *zAn*, and consequently *sn*, is diminished; and when the time of describing sn, by the diurnal motion, is 3′. 56″, the moon will then rise at the same solar hour. Let us suppose the latitude to be increased until the angle sAn vanishes, then the moon's orbit becomes coincident with the horizon every day, for a moment of time, and consequently the moon rises at the same sidereal hour, or 3′. 56″ sooner, by solar time. Now take a globe, and elevate the north pole to this latitude, and, marking the moon's orbit in this position upon it, turn the globe about, and it will appear, that at the instant after the above coincidence, one half of the moon's orbit, corresponding to Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini, will rise; hence, when the moon is going through that part of it's orbit, or for 13 or 14 days, it rises at the same sidereal hour. Now, taking the angle $xAE = 5°.\ 9'$, and the angle $EAQ = 23°.\ 28'$, the angle QAx, or QAH, when the moon's orbit coincides with the horizon, is 28°. 37′; hence, (87) the latitude is 61°. 23′ where these circumstances take place. If the descending node be at A, then x lying above E, QAx, or $QAH = 18°.\ 19'$, and the latitude is 71°. 41′. In any other situation of the orbit, the latitude will be between these limits. When the angle QAx is greater than the complement of latitude, the moon will rise sooner the next day. As there is a complete revolution of the nodes in about 18 years 8 months, all the varieties of the intervals of the rising and setting of the moon will happen within that time.

On the Horizontal Moon.

(235.) The phænomenon of the horizontal moon is this, that it appears larger in the horizon than in the meridian; whereas, from it's being nearer to us in the latter than in the former case, it subtends a greater angle. *Gassendus* thought that, as the moon was less bright in the horizon, we looked at it there with a greater pupil of the eye, and therefore it appeared larger. But this is contrary to the principles of Optics, since the magnitude of the image upon the retina does not depend upon the pupil. This opinion was supported by a French *Abbé*, who supposed that the opening of the pupil made the chrystalline humour flatter, and the eye longer, and thereby increased the image. But there is no connection between the muscles of the iris and the other parts of the eye, to produce these effects. *Des Cartes* thought that the moon appeared largest in the horizon, because, when comparing it's distance with the intermediate objects, it appeared then furthest off; and as we judge it's distance greatest in that situation, we of course think it larger, supposing that it subtends the same angle. This opinion was supported by Dr. *Wallis*, in the *Phil. Trans.* N°. 187. Dr. *Berkley* accounts for it thus. Faintness suggests the idea of greater distance; the moon appearing most faint in the horizon, suggests the idea of greater distance, and, supposing the visual angle the same, that must suggest the idea of a greater tangible object. He does not suppose the *visible* extension to be greater, but that the idea of a greater *tangible* extension is suggested, by the alteration of the appearance of the visible extension. He says, 1. That which suggests the idea of greater magnitude, must be something perceived; for what is not perceived can produce no visible effect. 2. It must be something which is variable, because the moon does not always appear of the same magnitude in the horizon. 3. It cannot lie in the intermediate objects, they remaining the same; also, when these objects are excluded from sight, it makes no alteration. 4. It cannot be the visible magnitude, because that is least in the horizon; the cause, therefore, must lie in the visible appearance, which proceeds from the greater paucity of rays coming to the eye, producing *faintness*. Mr. *Rowning* supposes that the moon appears furthest from us in the horizon, because the portion of the sky which we see, appears not an entire *hemisphere*, but only a portion of one; and in consequence of this, we judge the moon to be furthest from us in the horizon, and therefore to be then largest. Dr. *Smith*, in his *Optics*, gives the same reason. He makes the apparent distance in the horizon to be to that in the zenith as 10 to 3, and therefore the apparent diameters in that ratio. The methods by which he estimated the apparent distances, may be seen in Vol. I. page 65. The same circumstance also takes place in the sun, which appears much larger in the horizon than in the zenith. Also, if we take two stars near each other in the horizon, and two other stars near the zenith at the same angular distance from each other, the two former will appear at a much greater distance from each other, than the two latter. Upon this account, people are, in general, very much deceived in estimating the altitudes of the heavenly bodies above the horizon, judging them to be much greater than they are. Dr. *Smith* found, that when a body was about 23° above the horizon, it appeared to be half way between the zenith and horizon, and therefore at that real altitude it would be estimated to be 45° high. The lower part of a rainbow also appears broader than the upper part. And this may be considered as an argument that the phænomenon cannot depend entirely upon the greater degree of faintness in the object when in the horizon, because the lower part of the bow frequently appears brighter than the upper part, at the same time that it appears broader. Also, this cause could have no effect upon the distance of the stars; and as the difference of the apparent distance of the two stars, whose angular distance is the same, in the horizon and zenith, seems to be fully sufficient to account for the apparent variation of the moon's diameter in these situations, it may be doubtful, whether the faintness of the object enters into any part of the cause.

CHAP. XVII.

ON THE ROTATION OF THE SUN, AND PLANETS.

(236.) THE times of rotation of the sun, and planets, and the position of their axes, are determined from the spots which are observed upon their surfaces. The position of the same spot, observed at three different times, will give the position of the axis; for three points of any small circle will determine it's situation, and hence we know the axis of the sphere which is perpendicular to it. The time of rotation may be found, either from observing the arc of the small circle described by a spot in any time, or by observing the return of a spot to the same position in respect to the earth.

On the Rotation of the Sun.

(237.) It is doubtful by whom the spots on the sun were first discovered. *Scheiner*, Professor of Mathematics in Ingolstadt, observed them in May, 1611, and published an account of them in 1612, in a work entitled, *Rosa Ursina*. *Galileo*, in the Preface to a work entitled, *Istoria, Dimostrazioni, intorno alle Macchie Solari, Roma*, 1613, says, that being at Rome in 1611, he then showed the spots of the sun to several persons, and that he had spoken of them, some months before, to his friends at Florence. He imagined them to adhere to the sun. *Kepler*, in his Ephemeris, says, that they were observed by the son of *David Fabricius*, who published an account of them in 1611. In the papers of *Harriot*, not yet printed, it is said, that spots upon the sun were observed on December 8, 1610. As telescopes were in use at that time, it is probable that each might make the discovery. Admitting these spots to adhere to the sun's body, the reasons for which we shall afterwards give, we proceed to show how the time of it's rotation may be found.

(238.) M. *Cassini* determined the time of rotation, from observing the time in which a spot returns to the same situation upon the disc, or to the circle of latitude passing through the earth. Let t be that interval of time, and let m be equal to the *true* motion of the earth in that time, and n equal to it's *mean* motion; then $360° + m : 360° + n :: t :$ the time of return if the motion had been uniform, and this, from a great number of observations, he determines to be 27*d.* 12*h.* 20′; now the mean motion of the earth in that time is 27°. 7′. 8″; hence, 360° + 27°. 7′. 8″ : 360° :: 27*d.* 12*h.* 20′ : 25*d.* 14*h.* 8′, the time of rotation. *Elem. d'Astron.* p. 104.

(239.) When the earth is in the nodes of the sun's equator, and consequently in it's plane, the spots appear to describe straight lines: this happens about the beginning of June and December. As the earth recedes from the nodes, the path of a spot grows more and more elliptical, till the earth gets 90° from the nodes, which happens about the beginning of September and March, at which time the ellipse has it's minor axis the greatest, and is then to the major axis, as the sine of the inclination of the solar equator to radius.

(240.) There has been a great difference of opinions respecting the nature of the solar spots. *Scheiner* supposed them to be solid bodies revolving about the sun, very near to it; but as they are as long visible as they are invisible, this cannot be the case. Moreover, we have a physical argument against this hypothesis, which is, that most of them do not revolve about the sun in a plane passing through it's center, which they necessarily must, if they revolved, like the planets, about the sun. *Galileo* confuted *Scheiner*'s opinion, by observing that the spots were not permanent; that they varied their figure; that they increased, and sometimes disappeared. He compared them to smoak and clouds. *Hevelius* appears to have been of the same opinion; for in his *Cometographia*, p. 360, speaking of the solar spots, he says, *Hæc materia nunc ea ipsa est evaporatio et exhalatio (quia aliunde minime oriri potest) quæ ex ipso corpore solis, ut supra ostensum est, expiratur et exhalatur.* But the permanency of most of the spots is an argument against this hypothesis. M. *de la Hire* supposed them to be solid, opaque bodies, which swim upon the liquid matter of the sun, and which are sometimes entirely immersed. M. *de la Lande* supposes that the sun is an opaque body, covered with a liquid fire, and that the spots arise from the opaque parts, like rocks, which, by the alternate flux and reflux of the liquid igneous matter of the sun, are sometimes raised above the surface. The spots are frequently dark in the middle, with an umbra about them; and M. *de la Lande* supposes that the part of the rock which stands above the surface, forms the dark part in the center, and those parts which are but just covered by the igneous matter, form the umbra. Dr. *Wilson*, Professor of Astronomy at Glasgow, opposes this hypothesis of M. *de la Lande*, by this argument. Generally speaking, the umbra immediately contiguous to the dark central part, or nucleous, instead of being very dark, as it ought to be, from our seeing the immersed parts of the opaque rock through a thin stratum of the igneous matter, is, on the contrary, very nearly of the same splendour as the external surface, and the umbra grows darker the further it recedes from the nucleus; this, it must be acknowledged, is a strong argument against the hypothesis of M. *de la Lande.* Dr. *Wilson* further observes, that M. *de la Lande* produces no optical arguments in support of the rock standing above the surface of the sun. The opinion of Dr. *Wilson* is, that the spots are excavations in the luminous matter of the sun, the bottom of which forms the umbra. They who wish to see the arguments by which this is supported, must consult the *Phil. Trans.* 1774 and 1783. Dr. *Halley* conjectured that the spots are formed in the atmosphere of the sun. Dr. *Herschel* supposes the sun to be an opaque body, and that it has an atmosphere; and if some of the fluids which enter into it's composition should be of a shining brilliancy, whilst others are merely transparent, any temporary cause which may remove the lucid fluid will permit us to see the body of the sun through the transparent ones. See the *Phil. Trans.* 1795. Dr. *Herschel*, on April 19, 1779, saw a spot which measured 1′. 8″,06 in diameter, which is equal in length to more than 31000 miles; this was visible to the naked eye. Besides the dark spots upon the sun, there are also parts of the sun, called *Faculæ, Lucili*, &c. which are brighter than the general surface; these always abound most in the neighbourhood of the spots themselves, or where spots recently have been. Most of the spots appear within the compass of a zone lying 30° on each side of the equator; but on July 5, 1780, M. *de la Lande* observed a spot 40° from the equator. Spots which have disappeared, have been observed to break out again. The spots appear so frequently, that Astronomers very seldom examine the sun with their telescopes, but they see some; *Scheiner* saw fifty at once. The following phænomena of the spots are described by *Scheiner* and *Hevelius*.

I. Every spot which has a nucleus, has also an umbra surrounding it.

II. The boundary between the nucleus and umbra is always well defined.

III. The increase of a spot is gradual, the breadth of the nucleus and umbra dilating at the same time.

IV. The decrease of a spot is gradual, the breadth of the nucleus and umbra contracting at the same time.

V. The exterior boundary of the umbra never consists of sharp angles, but is always curvilinear, however irregular the outline of the nucleus may be.

VI. The nucleus, when on the decrease, in many cases changes it's figure, by the umbra encroaching irregularly upon it.

VII. It often happens, by these encroachments, that the nucleus is divided into two or more nuclei.

VIII. The nucleus vanishes sooner than the umbra.

IX. Small umbræ are often seen without nuclei.

X. An umbra of any considerable size is seldom seen without a nucleus.

XI. When a spot, consisting of a nucleus and umbra, is about to disappear, if it be not succeeded by a facula, or more fulgid appearance, the place it occupied is, soon after, not distinguishable from any other part of the sun's surface.

On the Rotation of the Planets.

(241.) The *Georgian* is at so great a distance, that Astronomers, with their best telescopes, have not been able to discover whether it has any revolution about it's axis.

(242.) *Saturn* was suspected by *Cassini* and *Fato*, in 1683, to have a revolution about it's axis; for they one day saw a bright streak, which disappeared the next, when another came into view near the edge of it's disc; these streaks are called *Belts*. In 1719, when the ring disappeared, *Cassini* saw it's shadow upon the body of the planet, and a belt on each side parallel to the shadow. When the ring was visible, he perceived the curvature of the belts was such as agreed with the elevation of the eye above the plane of the ring. He considered them as similar to our clouds floating in the air; and having a curvature similar to the exterior circumference of the ring, he concluded that they ought to be nearly at the same distance from the planet, and that consequently the atmosphere of Saturn extended to the ring. Dr. *Herschel* found that the arrangement of the belts always followed the direction of the ring; thus, as the ring opened, the belts began to show an incurvature answering to it. And during his observations on June 19, 20, and 21, 1780, he saw the same spot in three different situations. He conjectured, therefore, that Saturn revolved about an axis perpendicular to the plane of it's ring. Another argument in support of this is, that the planet is an oblate spheroid, having the diameter in the direction of the ring to the diameter perpendicular to it, as about 11 : 10, according to Dr. *Herschel*; the measures were taken with a wire micrometer prefixed to his 20 feet reflector. The truth of his conjecture he has now verified, having determined that Saturn revolves about it's axis in 10*h*. 16′. 0″,4. *Phil. Trans.* 1794. The rotation is according to the order of the signs.

(243.) *Jupiter* is observed to have belts, and also spots, by which the time of it's rotation can be very accurately ascertained. M. *Cassini* found the time of rotation to be 9*h*. 56′, from a remarkable spot which he observed in 1665. In October, 1691, he observed two bright spots almost as broad as the belts; and at the end of the month he saw two more, and found them to revolve in 9*h*. 51′; he also observed some other spots near Jupiter's equator, which revolved in 9*h*. 50′; and, in general, he found that the nearer the spots were to the equator, the quicker they revolved. It is probable, therefore, that the spots are not upon Jupiter's surface, but in it's atmosphere; and for this reason also, that several spots which appeared round at first, grew oblong by degrees in a direction parallel to the belts, and divided themselves into two or three spots. M. *Maraldi*, from a great many observations of the spot observed by *Cassini* in 1665, found the time of rotation to be 9*h*. 56′; and concluded that the spots had a dependence upon the contiguous belt, as the spot had never appeared without the belt, though the belt had without the spot. It continued to appear and disappear till 1694, and was not seen any more till 1708; hence, he concluded, that the spot was some effusion from the belt upon a fixed place of Jupiter's body, for it always appeared in the same place. Dr. *Herschel* found the time of rotation of different spots to vary; and that the time of rotation of the same spot diminished; for the spot observed in 1788 revolved as follows. From February 25 to March 2, in 9*h*. 55′. 20″; from March 2 to the 14th, in 9*h*. 54′. 58″; from April 7 to the 12th, in 9*h*. 51′. 35″. Also, from a spot observed in 1779, it's rotation was, from April 14 to the 19th, in 9*h*. 51′. 45″; from April 19 to the 23d, in 9*h*. 50′. 48″. This, he observes, is agreeable to the theory of equinoctial winds, as it may be some time before the spot can acquire the velocity of the wind; and if Jupiter's spots should be observed in different parts of it's revolution to be accelerated and retarded, it would amount almost to a demonstration of it's monsoons, and their periodical changes. M. *Schroeter* makes the time of rotation 9*h*. 55′. 36″,6; he observed the same variations as Dr. *Herschel*. The rotation is according to the order of the signs. This planet is observed to be flat at it's poles. Dr. *Pound* measured the polar and equatorial diameters, and found them as 12 : 13. Mr. *Short* made them as 13 : 14. Dr. *Bradley* made them as 12,5 : 13,5. Sir *I. Newton* makes the ratio $9\frac{1}{3} : 10\frac{1}{3}$ by theory. The belts of Jupiter are generally parallel to it's equator, which is very nearly parallel to the ecliptic; they are subject to very great variations, both in respect to their number and figure; sometimes eight have been seen at once, and at other times only one; sometimes they continue for three months without any variation, and sometimes a new belt has been formed in an hour or two. From their being subject to such changes, it is very probable that they do not adhere to the body of Jupiter, but exist in it's atmosphere.

(244.) *Galileo* discovered the phases of *Mars*; after which, some Italians in 1636, had an imperfect view of a spot. But in 1666, Dr. *Hook* and M. *Cassini* discovered some well-defined spots; and the latter determined the time of the rotation to be 24*h*. 40′. Soon after, M. *Maraldi* observed some spots, and determined the time of rotation to be 24*h*. 39′. He also observed a very bright part near the southern pole, appearing like a polar zone; this, he says, has been observed for 60 years; it is not of equal brightness, more than half of it being brighter than the rest; and that part which is least bright, is subject to great changes, and sometimes disappears. Something like this has been seen about the north pole. The rotation is according to the order of the signs. Dr. *Herschel* makes the time of a sidereal rotation to be 24*h*. 39′. 21″,67, without the probability of a greater error than 2″,34. He proposes to find the time of a sidereal rotation, in order to discover, by future observations, whether there is any alteration in the time of the revolution of the earth, or of the planets, about their axes; for a change of either would thus be discovered. He chose Mars, because it's spots are permanent. See the *Phil. Trans.* 1781. From further observations upon Mars, which he published in *Phil. Trans.* 1784, he makes it's axis to be inclined to the ecliptic 59°. 42′, and 61°. 18′ to it's orbit; and the north pole to be directed to 17°. 47′ of Pisces upon the ecliptic, and 19°. 28′ on it's orbit. He makes the ratio of the diameters of Mars to be as 16 : 15. Dr. *Maskelyne* has carefully observed Mars at the time of opposition, but could not perceive any difference in it's diameters. Dr. *Herschel* observes, that Mars has a considerable atmosphere.

(245.) *Galileo* first discovered the phases of *Venus* in 1611, and sent the discovery to *William de' Medici*, to communicate it to *Kepler*. He sent it in this cypher, *Hæc immaturæ a me frustra leguntur, o, y*; which put in order, is, *Cynthiæ figuras æmulatur mater amorum*, that is, *Venus emulates the phases of the moon*. He afterwards wrote a letter to him, giving an account of the discovery, and explaining the cypher. In 1666, M. *Cassini*, at a time when Venus was dichotomized, discovered a bright spot upon it at the straight edge, like some of the bright spots upon the moon's surface; and by observing it's motion, which was upon the edge, he found the sidereal time of rotation to be 23*h*. 16′. In the year 1726, *Bianchini* made some observations upon the spots of Venus, and asserted the time of rotation to be $24\frac{1}{3}$ days; that the north pole answered to the 20th degree of Aquarius, and was elevated from 15° to 20° above it's orbit; and that the axis continued parallel to itself. The small angle which the axis of Venus makes with it's orbit is a singular circumstance, and must cause a very great variety in the seasons. M. *Cassini*, the Son, has vindicated his Father, and shown, from *Bianchini*'s observations being interrupted, that he might easily mistake different spots for the same; and he concludes, that if we suppose the periodic time to be 23*h*. 20′, it agrees equally with their observations; but if we take it $24\frac{1}{3}$ days, it will not at all agree with his Father's observations. M. *Schroeter* has endeavoured to show that Venus has an atmosphere, from observing that the illuminated limb, when horned, exceeds a semicircle; this he supposes to arise from the refraction of the sun's rays through the atmosphere of Venus at the cusps, by which they appear prolonged. The cusps appeared sometimes to run 15°. 19′ into the dark hemisphere; from which he computes, that the height of the atmosphere, to refract such a quantity of light, must be 15156 Paris feet. But this must depend on the nature and density of the atmosphere, of which we are ignorant. *Phil. Trans.* 1792. He makes the time of rotation to be 23*h*. 21′, and concludes, from his observations, that there are considerable mountains upon this planet. *Phil. Trans.* 1795. Dr. *Herschel* agrees with M. *Schroeter*, that Venus has a considerable atmosphere; but he has not made any observations, by which he can determine, either the time of rotation, or the position of the axis. *Phil. Trans.* 1793.

(246.) The phases of *Mercury* are easily distinguished to be like those of *Venus*; but no spots have yet been discovered, by which we can ascertain whether it has any rotation.

(247.) The fifth satellite of *Saturn* was observed by M. *Cassini* for several years, as it went through the eastern part of it's orbit, to appear less and less, till it became invisible; and in the western part to increase again. These phænomena can hardly be accounted for, but by supposing some parts of the surface to be incapable of reflecting light, and therefore, when such parts are turned towards the earth, they appear to grow less, or to disappear. As the same appearances returned again when the satellite came to the same part of it's orbit, it affords an argument that the time of the rotation about it's axis is equal to the time of it's revolution about it's primary, a circumstance similar to the case of the moon and earth. See Dr. *Herschel*'s account of this in the *Phil. Trans.* 1792. The appearance of this satellite of Saturn is not always the same, and therefore it is probable that the dark parts are not permanent. Dr. *Herschel* has discovered that all the satellites of *Jupiter* have a rotatory motion about their axes, of the same duration with their respective periodic times about their primaries. *Phil. Trans.* 1797.

CHAP. XVIII.

ON THE SATELLITES.

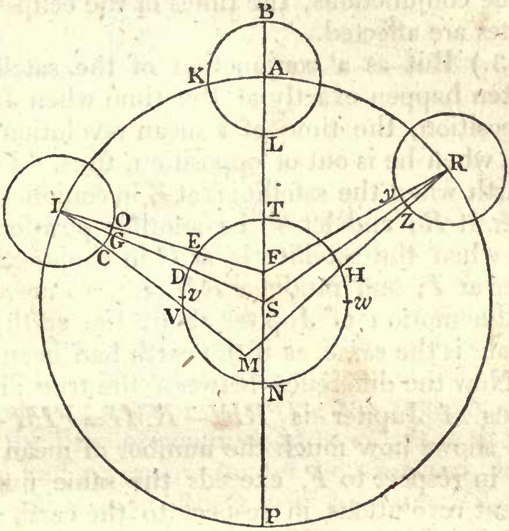
(248.) ON January 8, 1610, *Galileo* discovered the four satellites of *Jupiter*, and called them *Medicea Sidera*, or *Medicean Stars*, in honour of the family of the *Medici*, his patrons. This was a discovery, very important in it's consequences, as it furnished a ready method of finding the longitudes of places, by means of their eclipses; the eclipses led M. *Roemer* to the discovery of the progressive motion of light; and hence Dr. *Bradley* was enabled to solve an apparent motion in the fixed stars, which could not otherwise have been accounted for.

(249.) The satellites of *Jupiter*, in going from the west to the east, are eclipsed by the shadow of Jupiter, and as they go from east to west, they are observed to pass over it's disc; hence, they revolve about Jupiter, and in the same direction as Jupiter revolves about the sun. The three first satellites are always eclipsed, when they are in opposition to the sun, and the lengths of the eclipses are found to be different at different times; but sometimes the fourth satellite passes through opposition without being eclipsed. Hence it appears, that the planes of the orbits do not coincide with the plane of Jupiter's orbit, for, in that case, they would always pass through the center of Jupiter's shadow, and there would always be an eclipse, and of the same, or very nearly the same, duration, at every opposition to the sun. As the planes of the orbits which they describe sometimes pass through the eye, they will then appear to describe straight lines passing through the center of Jupiter; but at all other times they will appear to describe ellipses, of which Jupiter is the center.

On the Periodic Times, and Distances of Jupiter's Satellites.

(250.) To get the mean times of their *synodic* revolutions, or of their revolutions in respect to the sun, observe, when Jupiter is in opposition, the passage of a satellite over the body of Jupiter, and note the time when it appears to be exactly in conjunction with the center of Jupiter, and that will be the time of conjunction with the sun. After a considerable interval of time, repeat the same observation, Jupiter being in opposition, and divide the interval of time by the number of conjunctions with the sun in that interval, and you get the time of a *synodic* revolution of the satellite. This is the revolution which we have occasion principally to consider, it being that on which the eclipses depend. But, owing to the equation of Jupiter's orbit, this will not give the *mean* time of a synodic revolution, unless Jupiter was at the same point of it's orbit at both observations; otherwise, we must proceed thus.

(251.) Let *AIPR* be the orbit of Jupiter, *S* the sun in one focus, and *F* the other focus; and as the excentricity of the orbit is small, the motion about *F* may be considered (169) as uniform. Let Jupiter be in it's aphelion at *A* in opposition to the earth at *T*; and *L* a satellite in conjunction; and let *I* be the place of Jupiter at it's next opposition with the earth at *D*, and the satellite in conjunction at *G*. Then, if the satellite had been at *O*, it would have been in conjunction with *F*, or in mean conjunction; therefore it must describe the angle *FIS* before it comes to the mean conjunction, which angle is (169) the equation of the orbit, according to the *simple elliptic* *hypothesis*, which may be here used, as the excentricity of the orbit is but small; the angle *FIS* therefore measures the difference between the mean *synodic* revolutions in respect to *F*, and the synodic revolutions in



respect to the sun *S*. If, therefore, *n* be the number of revolutions which the satellite has made in respect to the sun, $n \times 360^\circ - SIF =$ the revolutions in respect to *F*; hence, $n \times 360^\circ - SIF : 360^\circ ::$ the time between the two oppositions : the time of a *mean* synodic revolution about the sun.

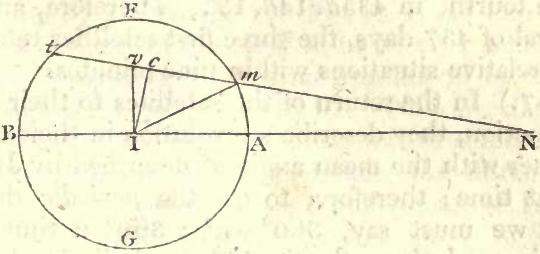
(252.) As the satellite is at *O* at the mean conjunction, and at *G* when in conjunction with the sun, it is manifest, that if the angle *FIS* continued the same, the time of a revolution in respect to *S* would be equal to the time in respect to *F*, or to the time of a mean synodic revolution; hence, the difference between the times of any two successive revolutions in respect to *S* and *F* respectively is as the variation of the angle *FIS*, or variation of the equation of the orbit. When Jupiter is at *A*, the equation vanishes, and the times of the two conjunctions at *F* and *S* coincide. When Jupiter comes to *I*, the mean conjunction at *O* happens after the true conjunction at *G*, by the time of describing the angle *SIF*, the equation of Jupiter's orbit. This is what Astronomers call the *first* inequality; and by this inequality of the intervals of the times of the true conjunctions, the times of the eclipses of the satellites are affected.

(253.) But as a conjunction of the satellite may not often happen exactly at the time when Jupiter is in opposition, the time of a mean revolution may be found, when he is out of opposition, thus. Let *H* be the earth when the satellite is at *Z* in conjunction with Jupiter at *R*; and let *V* be another position of the earth when the satellite is at *C* in conjunction with Jupiter at *I*; and produce *RH*, *IV*, to meet in *M*; then the motion of Jupiter about the earth, in this interval, is the same as if the earth had been fixed at *M*. Now the difference between the true and mean motions of Jupiter is $RFI - RMI = FIM + FRM$, which shows how much the number of mean revolutions, in respect to *F*, exceeds the same number of apparent revolutions in respect to the earth; hence, $n \times 360° - FIM - FRM : 360°$ :: the time between the observations : the time of a mean synodic revolution of the satellite. If *C* and *Z* lie on the other side of *O* and *Y*, the angles *FIM*, *FRM*, must be added to $n \times 360°$; and if one lie on one side, and the other on the other, one must be added and the other subtracted, according to the circumstances.

(254.) As it is difficult, from the great brightness of Jupiter, to determine accurately the time when the satellite is in conjunction with the center of Jupiter as it passes over it's disc, the time of conjunction is determined by observing it's entrance upon the disc, and it's going off; but as this cannot be determined with so much accuracy as the times of immersion into the shadow of Jupiter, and emersion from it, the time of conjunction can be most accurately determined from the eclipses.

(255.) Let *I* be the center of Jupiter's shadow *FG*,

Nmt the orbit of a satellite, N the node of the satellite's orbit upon the orbit of Jupiter; draw Iv perpendicular to IN, and Ic to Nt; and when the satellite comes to v, it is in conjunction* with the sun. Now both the immersion at m and emersion at t of the second, third, and fourth satellites may sometimes be observed, the middle point of time between which, gives the time of the middle of the eclipse at c; and by calculating cv, from knowing the angle N and NI, we get the time of conjunction at v. If both the immersion and emersion cannot be observed, take the time of either, and after a very long interval of time, when an eclipse happens as nearly as possible in the same situation in respect to the node, take the time of the same phænomenon, and from the interval of these times you will get the time of a revolution. By these different methods, M. *Cassini* found the times of the mean *synodic* revolutions of the four satellites to be as follows:



First.	*Second.*	*Third.*	*Fourth.*
$1^d.18^h.28'.36''$	$3^d.13^h.17'.54''$	$7^d.3^h.59'.36''$	$16^d.18^h.5'.7''$

(256.) Hence it appears, that 247 revolutions of the first satellite are performed in $437d.\ 3h.\ 44'$; 123 revolutions of the second, in 437*d*. 3*h*. 41′; 61 revolutions of the third, in 437*d*. 3*h*. 35′, and 26 revolutions of the fourth, in 435*d*. 14*h*. 13′. Therefore, after an interval of 437 days, the three first satellites return to their relative situations within nine minutes.

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* A satellite is said to be in conjunction, both when it is between the Sun and Jupiter, and when it is opposite to the Sun; the latter may be called superior, and the former inferior conjunction.

(257.) In the return of the satellites to their mean conjunction, they describe a revolution in their orbits, together with the mean angle a° described by Jupiter in that time; therefore, to get the *periodic* time of each, we must say, $360^\circ + a^\circ : 360^\circ ::$ time of a synodic revolution : the time of a periodic revolution; hence, the *periodic* times of each are;

First.	*Second.*	*Third.*	*Fourth.*
$1^d.18^h.27'.33''$	$3^d.13^h.13'.42''$	$7^d.3^h.42'.33''$	$16^d.16^h.32'.8''$

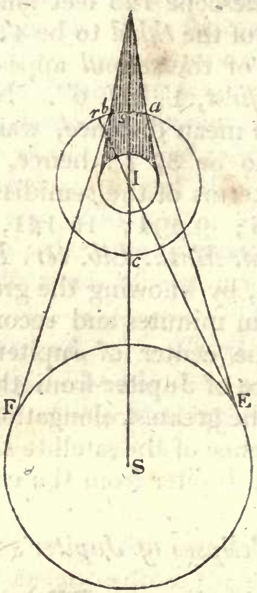
(258.) The distances of the satellites from the center of Jupiter may be found at the time of their greatest elongations, by measuring, with a micrometer, at that time, their distances from the center of Jupiter, and also the diameter of Jupiter, by which you get their distances in terms of the diameter. Or it may be done thus. When a satellite passes over the middle of the disc of Jupiter, observe the whole time of it's passage, and then, the time of a revolution : the time of it's passage over the disc :: 360° : the arc of it's orbit corresponding to the time of it's passage over the disc; hence, the sine of half that arc : radius :: the semidiameter of Jupiter : the distance of the satellite. Thus M. *Cassini* determined their distances in terms of the semidiameter of Jupiter to be, of the *first*, 5,67; of the *second*, 9; of the *third*, 14,38; and of the *fourth*, 25,3.

(259.) Or, having determined the periodic times, and the distance of one satellite, the distances of the other may be found from the proportion of the squares of the periodic times being as the cubes of their distances. Mr. *Pound*, with a telescope 15 feet long, found, at the mean distance of Jupiter from the earth, the greatest distance of the *fourth* satellite to be 8′. 16″; and by a telescope 123 feet long, he found the greatest distance of the *third* to be 4′. 42″; hence, the greatest distance of the *second* appears to be 2′. 56″. 47‴, and of the *first*, 1′. 51″. 6‴. Now the diameter of Jupiter, at it's mean distance, was determined, by Sir *I. Newton*, to be $37''\frac{1}{4}$; hence, the distances of the satellites, in terms of the semidiameter of Jupiter, come out 5,965; 9,494; 15,141, and 26,63 respectively. *Prin. Math. Lib. ter. Phœn.*

(260.) Hence, by knowing the greatest elongations of the satellites in minutes and seconds, we get their distances from the center of Jupiter, compared with the mean distance of Jupiter from the earth, by saying, the sine of the greatest elongation of the satellite : radius :: the distance of the satellite from Jupiter : the mean distance of Jupiter from the earth.

On the Eclipses of Jupiter's Satellites.

(261.) Let *S* be the sun, *EF* the orbit of the earth, *I* Jupiter, *abc* the orbit of one of it's satellites. When the earth is at *E* before the opposition of Jupiter, the spectator will see the immersion at *a*; but if it be the first satellite, upon account of it's nearness to Jupiter, the emersion is never visible, the satellite being then always behind the body of Jupiter; the other three satellites *may* have both their immersions and emersions visible; but this will depend upon the position of the earth. When the earth comes to *F* after opposition, we shall then see the emersion of the first, but can then never see the immersion; but we *may* see both the emersion and immersion of the other three. Draw *EIr*; then *sr*, the distance of the center of the shadow from the center of Jupiter, referred to the orbit of the satellite, is measured at Jupiter by *sr*, or the angle *sIr*, or the angle *EIS*. The satellite may be hidden behind the body at *r* without being eclipsed, which is called an *Occultation.* When the earth is at *E,* the conjunction of the satellite happens *later* at the earth than at the sun; but when the earth is at *F,* it happens *sooner.*



(262.) The diameter of the shadow of Jupiter, at the distance of any of the satellites, is best found by observing the time of an eclipse when it happens at the node, at which time the satellite passes through the center of the shadow; for the time of a synodic revolution : the time the satellite is passing through the center of the shadow :: 360° : the diameter of the shadow in degrees. But when the first and second satellites are in the nodes, the immersion and emersion cannot both be seen. Astronomers, therefore, compare the immersions some days *before* the opposition of Jupiter with the emersions some days *after,* and then, knowing how many synodic revolutions have been made, they get the time of the transit through the shadow, and thence the corresponding degrees. But on account of the excentricity of some of the orbits, the times of the central transit must vary; for example, the second satellite is sometimes found to be 2*h.* 50′ in passing through the center of the shadow, and sometimes 2*h.* 54′; this indicates an excentricity.

(263.) The duration of the eclipses being very unequal, shows that the orbits are inclined to the orbit of Jupiter; sometimes the fourth satellite passes through opposition without suffering an eclipse. The duration of the eclipses must depend upon the situation of the nodes in respect to the sun, just the same as in a lunar eclipse; when the line of the nodes passes through the sun, the satellite will pass through the center of the shadow; but as Jupiter revolves about the sun, the line of the nodes will be carried out of conjunction with the sun, and the time of the eclipse will be shortened, as the satellite will then describe only a chord of a section of the shadow instead of the diameter.

On the Rotation of the Satellites of Jupiter.

(264.) M. *Cassini* suspected that the satellites had a rotation about their axes, as sometimes in their passage over Jupiter's disc they were visible, and at other times not; he conjectured, therefore, that they had spots upon one side and not on the other, and that they were rendered visible in their passage when the spots were next to the earth. At different times also they appear of different magnitudes and of different brightness. The fourth appears generally the smallest, but sometimes the greatest; and the diameter of it's shadow on Jupiter appears sometimes greater than the satellite. The third also appears of a variable magnitude, and the like happens to the other two. Mr. *Pound* also observed, that they appear more luminous at one time than another, and therefore he concluded that they revolve about their axes. Dr. *Herschel* has discovered that they all revolve about their axes, in the times in which they respectively revolve about Jupiter.

On the Satellites of Saturn.

(265.) In the year 1655, *Huygens* discovered the fourth satellite of *Saturn*; and published a Table of it's mean motion in 1659. In 1671, M. *Cassini* discovered the fifth, and the third in 1672; and in 1684, the first and second; and afterwards published Tables of their motions. He called them *Sidera Lodoicea*, in honour of *Louis le Grand*, in whose reign, and observatory, they were discovered. Dr. *Halley* found, by his own observations in 1682, that *Huygens*'s Tables had considerably run out, they being about 15° in 20 years too forward, and therefore he composed new Tables from more correct elements. He also reformed M. *Cassini*'s Tables of the mean motions; and about the year 1720, published them a second time, corrected from Mr. *Pound*'s observations. He observes, that the four innermost satellites describe orbits very nearly in the plane of the ring, which, he says, is, as to the sense, parallel to the equator; and that the orbit of the fifth is a little inclined to them. The following Table contains the periodic times of the five satellites, and their distances in semidiameters of the ring, as determined by Mr. *Pound*, with a micrometer fitted to the telescope given by *Huygens* to the Royal Society. Mr. *Pound* first measured the distance of the fourth, and then deduced the rest from the proportion between the squares of the periodic times and cubes of their distances, and these are found to agree with observations.

Satellites.	Periodic Times by Pound.	Dist. in semid. of Ring, by Pound.	Dist. in semid. of Saturn by Pound.	Dist. in semid. of Ring by Cassini.	Dist. at the mean dist. of Saturn.
I	$1^{d}.21^{h}.18'.27''$	2,097	4,893	$1\frac{14}{15}$	$0'.43'',5$
II	2. 17. 41. 22	2,686	6,286	$2\frac{1}{2}$	0. 56
III	4. 12. 25. 12	3,752	8,754	$3\frac{1}{2}$	1. 18
IV	15.22.41.12	8,698	20,295	8	3. 0
V	79. 7. 49. 0	25,348	59,154	23	8. 42,5

The last column is from *Cassini*; but Dr. *Herschel* makes the distance of the fifth to be 8′. 31″,97, which is probably more exact. In this and the two next Tables, the satellites are numbered from Saturn as they were before the discovery of the other two.

On June 9, 1749, at 10*h*. Mr. *Pound* found the distance of the fourth satellite to be 3′. 7″ with a telescope of 123 feet, and an excellent micrometer fixed to it; and the satellite was at that time very near it's greatest eastern digression. Hence, at the mean distance of the earth from Saturn, that distance becomes 2′. 58″,21; Sir *I. Newton* makes it 3′. 4″.

(266.) The periodic times are found as for the satellites of Jupiter (251). To determine these, M. *Cassini* chose the time when the semi-minor axes of the eclipses which they describe, were the greatest, as Saturn was then 90° from their node, because the place of the satellite in it's orbit is then the same as upon the orbit of Saturn; whereas in every other case, it would be necessary to apply the reduction in order to get the place in it's orbit.

(267.) As it is difficult to see Saturn and the satellites at the same time in the field of view of a telescope, their distances have sometimes been measured, by observing the time of the passage of the body of Saturn over a wire adjusted as an hour circle in the field of the telescope, and the interval between the times when Saturn and the satellite passed. From comparing the periodic times and distances, M. *Cassini* observed that *Kepler's* Rule (162) agreed very well with observations.

(268.) By comparing the places of the satellites with the ring in different points of their orbits, and the greatest minor axes of the ellipses which they appear to describe, compared with the major axes, the planes of the orbits of the first four are found to be very nearly in the plane of the ring, and therefore are inclined to the orbit of Saturn about 30°; but the orbit of the fifth, according to M. *Cassini* the Son, makes an angle with the ring of about 15 degrees.

(269.) M. *Cassini* places the node of the ring, and consequently the nodes of the four first satellites, in 5^s. 22° upon the orbit of Saturn, and 5^s. 21° upon the ecliptic. M. *Huygens* had determined it to be in 5^s. 20°. 30′. M. *Maraldi*, in 1716, determined the longitude of the node of the ring upon the orbit of Saturn to be 5^s. 19°. 48′. 30″; and upon the ecliptic to be 5^s. 16°. 20′. The node of the fifth satellite is placed by M. *Cassini* in 5^s. 5° upon the orbit of Saturn. M. *de la Lande* makes it 5^s. 0°. 27′. From the observation of M. *Bernard*, at Marseilles, in 1787, it appears that the node of this satellite is retrograde.

(270.) Dr. *Halley* discovered that the orbit of the fourth satellite was excentric; for, having found it's mean motion, he discovered that it's place by observation was at one time 3° forwarder than by his calculations, and at other observations it was 2°. 30′ behind; this indicated an excentricity; and he placed the line of the apsides in 10^s. 22°. *Phil. Trans.* N°. 145.

Tables *of their* Revolutions *and* Mean Motions, *according to* M. de la Lande.

Satel.	*Diurnal Motion.*	*Motion in 365 days.*
I	6^s. 10°. 41′. 53″	4^s. 4°. 44′. 42″
II	4. 11. 32. 6	4. 10. 15. 19
III	2. 19. 41. 25	9. 16. 57. 5
IV	0. 22. 34. 38	10. 20. 39. 37
V	0. 4. 32. 17	7. 6. 23. 37

Satel.	*Periodic Revolution.*	*Synodic Revolution.*
I	1^d.21^h.18′.26″,222	1^d.21^h.18′.54″,778
II	2. 17. 44. 51,177	2. 17. 45. 51,013
III	4. 12. 25. 11,100	4. 12. 27. 55,239
IV	15. 22. 41. 16,022	15. 23. 15. 23,153
V	79. 7. 53. 42,772	79. 22. 3. 12,883

(271.) M. *Cassini* observed, that the fifth satellite disappeared regularly for about half it's revolution, when it was to the east of Saturn; from which he concluded, that it revolved about it's axis; he afterwards, however, doubted of this. But Sir *I. Newton*, in his *Principia*, Lib. III. Prop. 17, concludes from hence, that it revolves about it's axis, and in the same time that it revolves about Saturn; and that the variable appearance arises from some parts of the satellite not reflecting so much light as others. Dr. *Herschel* has confirmed this, by tracing regularly the periodical change of light through more than ten revolutions, which he found, in all appearances, to be cotemporary with the return of the satellite to the same situation in it's orbit. This is further confirmed by some observations of M. *Bernard*, at Marseilles, in 1787; and is a remarkable instance of analogy among the secondary planets.

(272.) These are all the satellites which were known to revolve about Saturn, till the year 1789, when Dr. *Herschel*, in a Paper in the *Phil. Trans.* for that year, announced the discovery of a *sixth* satellite, interior to all the others, and promised a further account in another Paper. But in the intermediate time he discovered a *seventh* satellite, interior to the sixth; and in a Paper upon Saturn and it's ring, in the *Phil. Trans.* 1790, he has given an account of the discovery, with some of the elements of their motions. He afterwards added Tables of their motions.

(273.) After his observations upon the ring, he says, he cannot quit the subject without mentioning his own surmises, and that of several other Astronomers, of a supposed roughness of the ring, or inequality in the planes and inclinations of it's flat sides. This supposition arose from seeing luminous points on it's boundaries, projecting like the moon's mountains; or from seeing one arm brighter or longer than the other; or even from seeing one arm when the other was invisible. Dr. *Herschel* was of this opinion, till he saw one of these points move off the edge of the ring in the form of a satellite. With his 20 feet telescope he suspected that he saw a sixth satellite; and on August 19, 1787, marked it down as probably being one; and having finished his telescope of 40 feet focal length, he saw six of it's satellites the moment he directed his telescope to the planet. This happened on August 28, 1789. The retrograde motion of Saturn was then nearly 4′. 30″ in a day, which made it very easy to ascertain whether the stars he took to be satellites, were really so; and in about two hours and an half after, he found that the planet had visibly carried them all away from their places. He continued his observations, and on September 17, he discovered the seventh satellite. These two satellites lie within the orbits of the other five. Their distances from the center of Saturn are 36″,7889, and 28″,6689; and their periodic times are 1*d.* 8*h.* 53′. 8″,9, and 22*h.* 37′. 22″,9. The planes of the orbits of these satellites lie so near to the plane of the ring, that the difference cannot be perceived.

On the Satellites of the Georgian.

(274.) On January 11, 1787, as Dr. *Herschel* was observing the *Georgian*, he perceived, near it's disc, some very small stars, whose places he noted. The next evening, upon examining them, he found that two of them were missing. Suspecting, therefore, that they might be satellites which had disappeared in consequence of having changed their situation, he continued his observations, and in the course of a month discovered them to be satellites, as he had first conjectured. Of this discovery he gave an account in the *Phil. Trans.* 1787.

(275.) In the *Phil. Trans.* 1788, he published a further account of this discovery, containing their periodic times, distances, and positions of their orbits, so far as he was then able to ascertain them. The most convenient method of determining the periodic time of a satellite, is, either from it's eclipses, or from taking it's position in several successive oppositions of the planet; but no eclipses have yet happened since the discovery of these satellites, and it would be waiting a long time to put in practice the other method. Dr. *Herschel*, therefore, took their situations whenever he could ascertain them with some degree of precision, and then reduced them, by computation, to such situations as were necessary for his purpose. In computing the periodic times, he has taken the synodic revolutions, as the positions of their orbits, at the times when their situations were taken, were not sufficiently known to get very accurate sidereal revolutions. The mean of several results gave the synodic revolution of the first satellite 8*d*. 17*h*. 1′. 19″,3, and of the second 13*d*. 11*h*. 5′. 1″,5. The results, he observes, of which these are a mean, do not much differ among themselves, and therefore the mean is probably tolerably accurate. The epochs from which their situations may, at any time, be computed are, for the *first*, Oct. 19, 1787, at 19*h*. 11′. 28″; and for the *second*, at 17*h*. 22′. 40″; at which times they were 76°. 43′ north, following the planet.

(276.) The next thing to be determined, in the elements of the satellites, was their distances from the planet; to obtain which, he found one distance by observation, and then the other from the periodic times (Article 162). Now, in attempting to discover the distance of the second, the orbit was seemingly elliptical. On March 18, 1787, at 8*h*. 2′. 50″, he found the elongation to be 46″,46, this being the greatest of all the measures he had taken. Hence, at the mean distance of the Georgian from the earth, this elongation will be 44″,23. Admitting, therefore, for the present, says Dr. *Herschel*, that the satellites move in circular orbits, we may take 44″,23 for the true distance, without much error; hence, as the squares of the periodic times are as the cubes of the distances, the distance of the first satellite comes out 33″,09. The synodic revolutions were here used instead of the sidereal, which will make but a small error.

(277.) The last thing to be done, was to determine the inclinations of the orbits, and places of their nodes. And here a difficulty presented itself which could not be got over at the time of his first observation; for it could not then be determined which part of the orbit was inclined *to* the earth, and which *from* it. On the two different suppositions, therefore, Dr. *Herschel* has computed the inclinations of the orbits, and the places of their nodes, and found them as follows. The orbit of the second satellite is inclined to the ecliptic 99°. 43′. 53″,3, or 81°. 6′. 4″,4; it's ascending node upon the ecliptic is in 5^s. 18°, or 8^s. 6°; and when the planet comes to the ascending node of this satellite, which will happen about the year 1818, the northern half of the orbit will be turned towards the east or west, at the time of it's meridian passage. M. *de la Lambre* makes the ascending node in 5^s. 21°, or 8^s. 9°, from Dr. *Herschel*'s observations. The situation of the orbit of the first satellite does not materially differ from that of the second. The light of the satellites is extremely faint; the second is the brightest, but the difference is small. The satellites are probably not less than those of *Jupiter*. There will be eclipses of these satellites about the year 1818, when they will appear to ascend through the shadow of the planet, in a direction almost perpendicular to the ecliptic.

Since these discoveries were made, Dr. *Herschel* has discovered four more satellites of the Georgian, and found that their motions are all retrograde. *Phil. Trans.* 1798.

Chap. XIX.

ON THE RING OF SATURN.

(278.) Galileo was the first person who observed any thing extraordinary in *Saturn*. The planet appeared to him like a large globe between two small ones. In the year 1610 he announced this discovery. He continued his observations till 1612, when he was surprised to find only the middle globe; but some time after he again discovered the globes on each side, which, in process of time, appeared to change their form; sometimes appearing round, sometimes oblong like an acorn, sometimes semicircular, then with horns towards the globe in the middle, and growing, by degrees, so long and wide as to encompass it, as it were, with an oval ring. Upon this, *Huygens* set about improving the art of grinding object glasses; and made telescopes which magnified two or three times more than any which had been before made, with which he discovered very clearly the ring of Saturn; and having observed it for some time, he published the discovery in 1656. He made the space between the globe and the ring equal to, or rather bigger than the breadth of the ring; and the greater diameter of the ring to that of the globe as 9 to 4. But Mr. *Pound*, with a micrometer applied to *Huygens*'s telescope of 123 feet long, determined the ratio to be as 7 to 3. Mr. *Whiston*, in his Memoirs of the Life of Dr. *Clark*, relates, that the Doctor's Father once saw a fixed star between the ring and the body of Saturn. In the year 1675, M. *Cassini* saw the ring, and observed upon it a dark elliptical line, dividing it, as it were, into two rings, the inner of which appeared brighter than the outer. He also observed a dark belt upon the planet, parallel to the major axis of the ring. Mr. *Hadley* observed that the outer part of the ring seemed narrower than the inner part, and that the dark line was fainter towards it's upper edge; he also saw two belts, and observed the shadow of the ring upon Saturn. In October, 1714, when the plane of the ring very nearly passed through the earth, and was approaching it, M. *Maraldi* observed, that while the arms were decreasing both in length and breadth, the eastern arm appeared a little larger than the other for three or four nights, and yet it vanished first, for, after two nights interruption by clouds, he saw the western arm alone. This inequality of the ring made him suspect that it was not bounded by exactly parallel planes, and that it turned about it's axis. But the best description of this singular phænomenon is that given by Dr. *Herschel*, in the *Phil. Trans.* 1790, who, by his extraordinary telescopes, has discovered many circumstances which had escaped all other observers. We shall here give the substance of his account.

(279.) The black disc, or belt, upon the ring of Saturn, is not in the middle of it's breadth; nor is the ring subdivided by many such lines, as has been represented by some Astronomers; but there is one* single, dark, considerable broad line, belt, or zone, which he has constantly found on the north side of the ring. As this dark belt is subject to no change whatever, it is probably owing to some permanent construction of the surface of the ring. This construction cannot be owing to the shadow of a chain of mountains, since it is visible all round on the ring;

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* In a Paper in the *Phil. Trans.* 1792, Dr. *Herschel* observes, that, "since the year 1774 to the present time, I can find only four observations where any other black division of the ring is mentioned, than the one which I have constantly observed; these were all in June, 1780."

for at the ends of the ring there could be no shade; and the same argument will hold against any supposed caverns. It is moreover pretty evident, that this dark zone is contained between two concentric circles, as all the phænomena answer to the projection of such a zone. The matter of the ring is undoubtedly no less solid than the planet itself; and it is observed to cast a strong shadow upon the planet. The light of the ring is also generally brighter than that of the planet; for the ring appears sufficiently bright, when the telescope affords scarcely light enough for Saturn. Dr. *Herschel* next takes notice of the extreme thinness of the ring. He frequently saw the first, second, third, fourth, and fifth satellites pass before and behind the ring, in such a manner that they served as an excellent micrometer to measure it's thickness by. It may be proper to mention a few instances, as they serve also to solve some phænomena observed by other Astronomers, without having been accounted for in any manner that could be admitted consistently with other known facts. July 18, 1789, at 16*h*. 41′. 9″ sidereal time, the third satellite seemed to hang upon the following arm, declining a little towards the north, and was seen gradually to advance upon it towards the body of Saturn; but the ring was not so thick as the lucid point. July 23, at 19*h*. 41′. 8″, the fourth satellite was a very little preceding the ring, but the ring appeared to be less than half the thickness of the satellite. July 27, at 20*h*. 15′. 12″, the fourth satellite was about the middle, upon the following arm of the ring, and towards the south; and the second at the farther end, towards the north; but the arm was thinner than either. August 29, at 22*h*. 12′. 25″, the fifth satellite was upon the ring, near the end of the preceding arm, and the thickness of the arm seemed to be about $\frac{1}{3}$ or $\frac{1}{4}$ of the diameter of the satellite, which, in the situation it then was, he took to be less than one-second in diameter. At the same time, the first appeared at a little distance following the fifth, in the shape of a bead upon a thread, projecting on both sides of the same arm; hence, the arm is thinner than the first, which is considerably smaller than the second, and a little less than the third. October 16, he followed the first and second satellites up to the very disc of the planet; and the ring, which was extremely faint, did not obstruct his seeing them gradually approach the disc. These observations are sufficient to show the extreme thinness of the ring. But Dr. *Herschel* further observes, that there may be a refraction through an atmosphere of the ring, by which the satellites may be lifted up and depressed, so as to become visible on both sides of the ring, even though the ring should be equal in thickness to the smallest satellite, which may amount to 1000 miles. From a series of observations upon luminous points of the ring, he has discovered that it has a rotation about it's axis, the time of which is 10*h*. 32′. 15″4.

(280.) The ring is invisible * when it's plane passes through the sun, or the earth, or between them; in the first case, the sun shines only upon it's edge, which is too thin to reflect sufficient light to render it visible; in the second case, the edge only being opposed to us, it is not visible, for the same reason; in the third case, the dark side of the ring is exposed to us, and therefore the edge being the only luminous part which is towards the earth, it is invisible on the same account as before. Observers have differed 10 or 12 days in the time of it's becoming invisible, owing to the difference of the telescopes, and of the state of the atmosphere. Dr. *Herschel* observes, that the ring was seen in his telescope, when we were turned towards the unenlightened side; so that he either saw the light reflected from the edge, or else the reflection of the light of Saturn upon the dark side of the ring, as we sometimes see the dark part of the moon. He cannot, however, say which of the two might be the case; especially as there are very strong reasons to think, that the edge of the ring is of such a nature as not to reflect much light. M. *de la Lande* thinks that the ring is just visible with the best telescopes in common use, when the sun is elevated 3′ above it's plane, or three days before it's plane passes through the sun; and when the earth is elevated 2′. 20″ above the plane, or one day from the earth's passing it.

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* The disappearance of the ring is only with the telescopes in common use among Astronomers; for Dr. *Herschel*, with his large telescopes, has been able to see it in every situation. He thinks the edge of the ring is not flat, but spherical, or spheroidical.

(281.) In a paper in the *Phil. Trans.* 1790, Dr. *Herschel* ventured to hint at a suspicion that the ring was divided; this conjecture was strengthened by future observations, after he had an opportunity of seeing both sides of the ring. His reasons are these: 1. The black division, upon the southern side of the ring, is in the same place, of the same breadth, and at the same distance from the outer edge, that it always appeared upon the northern side. 2. With his seven feet reflector and an excellent speculum, he saw the division on the ring, and the open space between the ring and the body, equally dark, and of the same colour with the heavens about the planet. 3. The black division is equally broad on each side of the ring. From these observations, Dr. *Herschel* thinks himself authorised to say, that Saturn has two concentric rings, situated in one plane, which is probably not much inclined to the equator of the planet. The dimensions of the rings are in the following proportions, as nearly as they could be ascertained.

	Parts.
Inside diameter of the smaller ring -	5900
Outside diameter - - - - - - -	7510
Inside diameter of the larger ring -	7740
Outside diameter - - - - - - -	8300
Breadth of the inner ring - - - -	805
Breadth of the outer ring - - - -	280
Breadth of the space between the rings	115

In the *Mem. de l'Acad.* at Paris, 1787, M. *de la Place* supposes that the ring may have many divisions; but Dr. *Herschel* remarks, that no observations will justify this supposition.

(282.) From the mean of a great many measures of the diameter of the larger ring, Dr. *Herschel* makes it 46″,677 at the mean distance of Saturn. Hence, it's diameter : the diameter of the earth :: 25,8914 : 1. From the above proportion, therefore, the diameter of the ring must be 204883 miles; and the distance of the two rings 2839 miles.

(283.) The ring being a circle, appears elliptical from it's oblique position; and it appears most open when Saturn is 90° from the nodes of the ring upon the orbit of Saturn, or when Saturn's longitude is about 2^{s}. 17°, and 8^{s}. 17°. In such a situation, the minor axis is extremely nearly equal to half the major, when the observations are reduced to the sun; consequently the plane of the ring makes an angle of about 30° with the orbit of Saturn.

CHAP. XX.

ON THE ABERRATION OF LIGHT.

(284.) IN the year 1725, Mr. *Molyneux*, assisted by Dr. *Bradley*, fitted up a zenith sector at Kew, in order to discover whether the fixed stars had any sensible parallax*, that is, whether a star would appear to have changed it's place whilst the earth moved from one extremity of the diameter of it's orbit to the other; or, which is the same, to determine whether the diameter of the earth's orbit subtends any sensible angle at the star. The very important discovery arising from their observations is so clearly and fully related by Dr. *Bradley*, in a letter to Dr. *Halley*, that I cannot do better than give it to the reader in his own words. *Phil. Trans.* N°. 406.

(285.) " Mr. *Molyneux*'s apparatus was completed and fitted for observing, about the end of November, 1725, and on the third day of December following, the bright star in the head of *Draco*, marked γ by *Bayer*, was for the first time observed as it passed near the zenith, and it's situation carefully taken with the instrument. The like observations were made on the 5th, 11th, and 12th of the same month; and there appearing no material difference in the place of the star, a farther repetition of them at this season seemed needless, it being a part of the year wherein no sensible alteration of parallax in this star could soon be expected. It was chiefly, therefore, curiosity that tempted me (being then at Kew, where the instrument was fixed) to prepare for observing the star on December 17, when, having adjusted the instrument as usual, I perceived that it passed a little more southwardly this day than when it was observed before. Not suspecting any other cause of this appearance, we first concluded that it was owing to the uncertainty of the observations, and that either this or the foregoing were not so exact as we had supposed; for which reason we purposed to repeat the observation again, in order to determine from whence this difference proceeded; and upon doing it on December 20, I found that the star passed still more southwardly than in the former observations. This sensible alteration the more surprized us, in that it was the contrary way from what it would have been, had it proceeded from an annual parallax of the star: but being now pretty well satisfied that it could not be entirely owing to the want of exactness in the observations, and having no notion of any thing else that could cause such an apparent motion as this in the star, we began to think that some change of the materials, &c. of the instrument itself might have occasioned it. Under these apprehensions we remained some time; but being at length fully convinced, by several trials, of the great exactness of the instrument, and finding, by the gradual increase of the star's distance from the pole, that there must be some regular cause that produced it, we took care to examine nicely, at the time of each observation, how much it was; and about the beginning of March, 1726, the star was found to be 20″ more southwardly than at the time of the first observation. It now, indeed, seemed to have arrived at it's utmost limit southward, because in several trials made about this time, no sensible difference was observed it's situation. By the middle of April it appeared to be returning back again towards the north; and about the beginning of June it passed at the same distance from the zenith as it had done in December when it was first observed.

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* Dr. *Hook* was the first inventor of this method, and in the year 1669 he put it in practice at Gresham College, with a telescope 36 feet long. His first observation was July 6, at which time he found the bright star in the head of *Draco*, marked γ by *Bayer*, passed about 2'. 12" northward from the zenith; on July 9, it passed at the same distance; on August 6, it passed 2'. 6" northward from the zenith; on October 21, it passed 1'. 48" or 50" north from the zenith, according to his observations. See his *Cutlerian* Lectures.

From the quick alteration of this star's declination about this time (it increasing a second in three days) it was concluded that it would now proceed northward, as it before had gone southward of it's present situation; and it happened as was conjectured, for the star continued to move northward till September following, when it again became stationary, being then near 20″ more northwardly than in June, and no less than 39″ more northwardly than it was in March. From September the star returned towards the south, till it arrived in December to the same situation it was in at that time twelve months, allowing for the difference of declination on account of the precession of the equinox.

This was a sufficient proof that the instrument had not been the cause of this apparent motion of the star, and to find one adequate to such an effect, seemed a difficulty. A nutation of the earth's axis was one of the first things that offered itself upon this occasion, but it was soon found to be insufficient; for though it might have accounted for the change of declination in γ Draconis, yet it would not at the same time agree with the phænomena in other stars, particularly in a small one almost opposite in right ascension to γ Draconis, at about the same distance from the north pole of the equator; for though this star seemed to move the same way as a nutation of the earth's axis would have made it, yet it changing it's declination but about half as much as γ Draconis in the same time, (as appeared upon comparing the observations of both made upon the same days at different seasons of the year,) this plainly proved that the apparent motion of the stars was not occasioned by a real nutation, since, if that had been the cause, the alteration in both stars would have been nearly equal.

The great regularity of the observations left no room to doubt but that there was some regular cause that produced this unexpected motion, which did not depend on the uncertainty or variety of the seasons of the year. Upon comparing the observations with each other, it was discovered, that in both the fore-mentioned stars, the apparent difference of declination from the maxima was always nearly proportional to the versed sine of the sun's distance from the equinoctial points. This was an inducement to think that the cause, whatever it was, had some relation to the sun's situation with respect to those points. But not being able to frame any hypothesis at that time, sufficient to solve all the phænomena, and being very desirous to search a little farther into this matter, I began to think of erecting an instrument for myself at Wansted; that, having it always at hand, I might with the more ease and certainty enquire into the laws of this new motion. The consideration likewise, of being able, by another instrument, to confirm the truth of the observations hitherto made with Mr. *Molyneux*'s, was no small inducement to me; but the chief of all was, the opportunity I should thereby have of trying in what manner other stars were affected by the same cause, whatever it was. For Mr. *Molyneux*'s instrument being originally designed for observing γ Draconis (in order, as I said before, to try whether it had any sensible parallax) was so contrived as to be capable of but little alteration in it's direction, not above seven or eight minutes of a degree; and there being few stars within half that distance from the zenith of Kew, bright enough to be well observed, he could not with his instrument thoroughly examine how this cause affected stars differently situated with respect to the equinoctial and solstitial points of the ecliptic.

These considerations determined me; and by the contrivance and direction of the very ingenious person Mr. *Graham*, my instrument was fixed up August

19, 1727. As I had no convenient place where I could make use of so long a telescope as Mr. *Molyneux*'s, I contented myself with one of but little more than half the length of his (viz. of about 12½ feet, his being 24¼) judging, from the experience which I had already had, that this radius would be long enough to adjust the instrument to a sufficient degree of exactness, and I have had no reason since to change my opinion; for, from all the trials I have yet made, I am well satisfied, that when it is carefully rectified, it's situation may be securely depended upon to half a second. As the place where my instrument was to be hung, in some measure determined it's radius, so did it also the length of the arch or limb on which the divisions were made to adjust it; for the arch could not conveniently be extended farther than to reach to about 6°. 15′ on each side my zenith. This indeed was sufficient, since it gave an opportunity of making choice of several stars very different both in magnitude and situation, there being more than two hundred inserted in the British Catalogue, that may be observed with it. I needed not to have extended the limb so far, but that I was willing to take in Capella, the only star of the first magnitude that comes so near to my zenith.

My instrument being fixed, I immediately began to observe such stars as I judged most proper to give me light into the cause of the motion already mentioned. There was variety enough of small ones, and not less than twelve that I could observe through all the seasons of the year, they being bright enough to be seen in the day-time, when nearest the sun. I had not been long observing, before I perceived that the notion we had before entertained, of the stars being farthest north and south when the sun was about the equinoxes, was only true of those that were near the solstitial colure; and after I had continued my observations a few months, I discovered, what I then apprehended to be a general law, observed by all the stars, viz. that each of them became stationary, or was farthest north or south when they passed over my zenith at six o'clock either in the morning or evening. I perceived likewise, that whatever situation the stars were in, with respect to the cardinal points of the ecliptic, the apparent motion of every one tended the same way when they passed my instrument about the same hour of the day or night; for they all moved southward while they passed in the day, and northward in the night; so that each was farthest north when it came about six o'clock in the evening, and farthest south when it came about six in the morning.

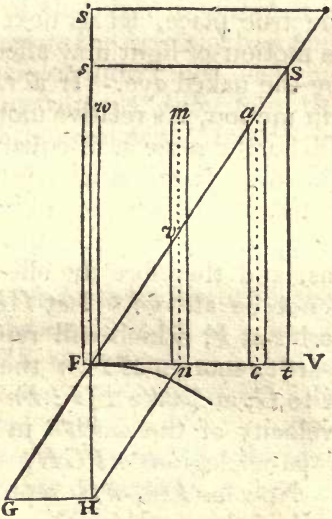
Though I have since discovered that the maxima in most of these stars do not happen exactly when they come to my instrument at those hours, yet, not being able at that time to prove the contrary, and supposing that they did, I endeavoured to find out what proportion the greatest alterations of declination in different stars bore to each other; it being very evident that they did not all change their declinations equally. I have before taken notice that it appeared from Mr. *Molyneux*'s observations that γ Draconis altered it's declination about twice as much as the fore-mentioned small star almost opposite to it; but examining the matter more particularly, I found that the greatest alteration of declination in these stars was as the sine of the latitude of each respectively. This made me suspect, that there might be the like proportion between the maxima of other stars; but finding that the observations of some of them would not perfectly correspond with such an hypothesis, and not knowing whether the small difference I met with might not be owing to the uncertainty and error of the observations, I deferred the farther examination into the truth of this hypothesis, till I should be furnished with a series of observations made in all parts of the year; which might enable me, not only to determine what errors the observations are liable to, or how far they may safely be depended upon; but to judge whether there had been any sensible change in the parts of the instrument itself.

Upon these considerations I laid aside all thoughts at that time about the cause of the fore-mentioned phænomena, hoping that I should the easier discover it, when I was better provided with proper means to determine more precisely what they were.

When the year was completed, I began to examine and compare my observations, and having pretty well satisfied myself as to the general laws of the phænomena, I then endeavoured to find out the cause of them. I was already convinced, that the apparent motion of the stars was not owing to a nutation of the earth's axis. The next thing that offered itself, was an alteration in the direction of the plumb-line with which the instrument was constantly rectified; but this, upon trial, proved insufficient. Then I considered what refraction might do; but here also nothing satisfactory occurred. At last I conjectured, that all the phænomena hitherto mentioned, proceeded from the progressive motion of light and the earth's annual motion in it's orbit. For I perceived, if light was propagated in time, the apparent place of a fixed object would not be the same when the eye is at rest, as when it is moving in any other direction than that of the line passing through the eye and object; and that when the eye is moving in different directions, the apparent place of the object would be different."

This is Dr. *Bradley*'s account of this very important discovery; we shall therefore proceed to show that his principle will solve all the phænomena.

(286.) The situation of any object in the heavens is determined by the position of the axis of the telescope annexed to the instrument with which we measure; for such a position is given to the telescope, that the rays of light from the object may descend down the axis, and in that situation the index shows the angular distance required. Now if light be progressive, when a ray from any object descends down the axis, the position of the telescope must be different from what it would have been, if light had been instantaneous, and therefore the place to which the telescope is directed, will be different from the true place of the object. For let S' be a fixed star, VF the direction of the earth's motion, $S'F$ the direction of a particle of light, entering the axis ac of a telescope at a, and moving through aF while the earth moves from c to F; then, if the telescope continue parallel to itself, the light will descend in the axis. For let the axis nm, Fw, continue parallel to ac; then, considering each motion * as uniform, the spaces described in the same time will continue in the same proportion; but $cF : aF :: cn : av$, and by supposition cF, aF, are described



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* The motion of the spectator arising from the rotation of the earth about it's axis is not here taken into consideration, it being so small as not to produce any sensible effect.

in the same time, therefore *cn*, *av*, are described in the same time; hence, when the telescope comes into the situation *nm*, the particle of light will be in the axis at *v*; and this being true for every instant, in this position of the telescope the ray will descend down the axis, and consequently the place of the star, determined by the telescope at *F*, is *s′*, and the angle *S′Fs′* is the *aberration*, or *the difference between the true place of the star and the place determined by the instrument*. Hence, if we take any line *FS* : *Ft* :: velocity of light : the velocity of the earth, and join *St*, and complete the parallelogram *FtSs*, the aberration will be equal to the angle *FSt*. Also, *S* represents the true place of the star, and *s* the place determined by the instrument.

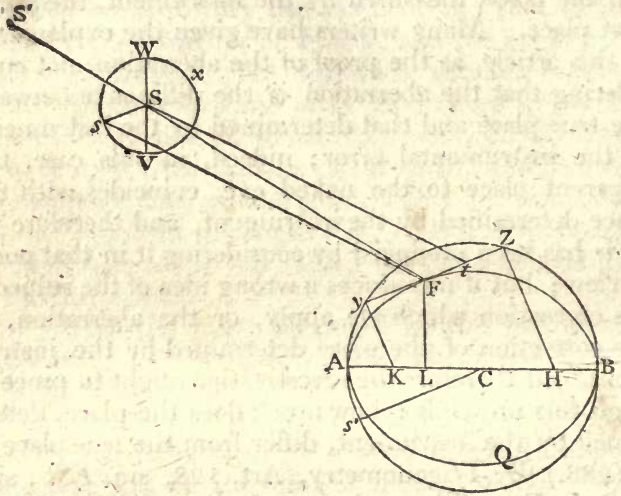
(287.) As the place measured by the instrument differs from the true place, let us next consider how the progressive motion of light may affect the place of the star seen by the naked eye. If a ray of light fall upon the eye in motion, it's relative motion, in respect to the eye, will be the same as if equal motions were impressed in the same direction upon each, at the instant of contact; for equal motions in the same direction, impressed upon two bodies, will not affect their relative motions, and therefore the effect of one upon the other will not be altered. Let *VF* be a tangent to the earth's orbit at *F*, which will represent the direction of the earth's motion at *F*, *S′* the star, join *S′F*, and produce it to *G*, and take *FG* : *Fn* :: the velocity of light : the velocity of the earth's in it's orbit, and complete the parallelogram *nFGH*, and draw the diagonal *FH*. Now as *FG*, *nF*, represent the motions of light and of the earth in it's orbit, conceive a motion *Fn* equal and opposite to *nF* to be impressed upon the eye at *F*, and upon the ray of light, then the eye will be at rest, and the ray of light, by the two motions *FG*, *Fn*, will describe the diagonal *FH*; this, therefore, is the relative motion of the ray of light in respect to the eye itself. Hence, the object appears in the direction *HF*, and consequently it's apparent place differs from it's true place by the angle $GFH = FSt$. It appears, therefore, that the apparent place of the object to the naked eye, is the same as the place determined by the instrument. We may therefore call the place, measured by the instrument, the apparent place. Many writers have given the explanation in this article, as the proof of the aberration, not considering that the aberration is the difference between the true place and that determined by the instrument, or the instrumental error; indeed, in this case, the apparent place to the naked eye, coincides with the place determined by the instrument, and therefore no error has been produced by considering it in that point of view; but it introduces a wrong idea of the subject; the correction which we apply, or the aberration, is the correction of the place determined by the instrument, and therefore the investigation ought to proceed upon this principle; how much does the place, determined by the instrument, differ from the true place?

(288.) By Trigonometry, Art. 128, sin. FSt : sin. FtS :: Ft : FS :: velocity of the earth : velocity of light; hence, sine of aberration = sin. $FtS \times \frac{\text{vel. of earth}}{\text{vel. of light}}$; therefore, if we consider the velocity of the earth and of light to be constant, the sine of aberration, or the aberration itself, as it never exceeds 20″, varies as sin. FtS, and therefore is greatest when that angle is a right angle; if, therefore, s be put for the sine of FtS, we have 1 (rad.) : s :: 20″ : $s \times 20''$ the aberration. Hence, when Ft coincides with FS', or the earth is moving directly to or from a star, there is no aberration.

(289.) As (by observation) the angle $FSt = 20''$, when $FtS = 90°$, we have, *the velocity of the earth* : *velocity of light* :: *sin.* 20″ : *radius* :: 1 : 10314.

(290.) The aberration $S's'$ lies, from the true place of the star, in a direction parallel to the direction of the earth's motion, and towards the same part.

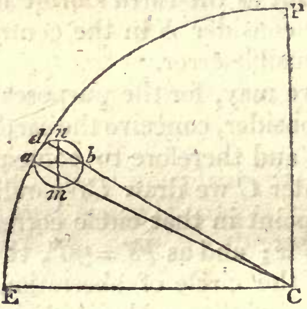
(291.) Whilst the earth makes one revolution in it's orbit, the curve, parallel to the ecliptic, described by the apparent place of a fixed star, is a circle. For let *AFBQ* be the earth's orbit, *K* the focus in which the sun is, *H* the other focus; on the major axis *AB* describe a circle in the same plane; draw a tangent *yFZ* to the point *F*, and *Ky*, *HZ*, perpendicular to it; then (Conic Sect. Ellipse, prop. 5), the points *y* and *Z* will be always in the circumference of the circle. Let *S*′ be the true place of the star, any where out of the plane of the ecliptic, which therefore must be conceived as elevated above the plane *AFBQ*, and take *tF* : *FS* as the velocity of the earth to the velocity of light, and complete the parallelogram *FtSs*, and *s* will (286) be the apparent place of the star. Draw *FL* perpendicular to *AB*, and let *WsVx* be the curve described by the point *s*, and *WSV* be parallel to *FL*. Now (from physical principles) the velocity of the earth varies as $\frac{1}{Ky}$, or as *HZ* (Con. Sec. El. p. 6); but *tF*, or *Ss*, represents the velocity of the earth; hence, *Ss* varies as *HZ*. Also, as *Ss*, *SV*, are parallel to *Fy*, *FL*, the



angle sSV = the angle yFL, which is = the angle ZHL, because the angle LFZ added to each makes two right angles, for in the quadrilateral figure $LFZH$, the angles L and Z are right ones. Hence, as Ss varies as HZ, and the angle $sSV = ZHA$, the figures described by the points s and Z must be similar; but Z describes a circle in the time of one revolution of the earth in it's orbit; hence, s must describe a circle about S in the same time. And as Ss is always parallel to tF which lies in the plane of the ecliptic, the circle $WsVx$ is parallel to the ecliptic. Also, as S and H are two points similarly situated in WV and AB, it appears that the true place of the star divides that diameter, which, although in a different plane, we may consider as perpendicular to the major axis of the earth's orbit, in the same ratio as the focus divides the major axis. But as the earth's orbit is very nearly a circle, we may consider S in the centre of the circle, without any sensible error.

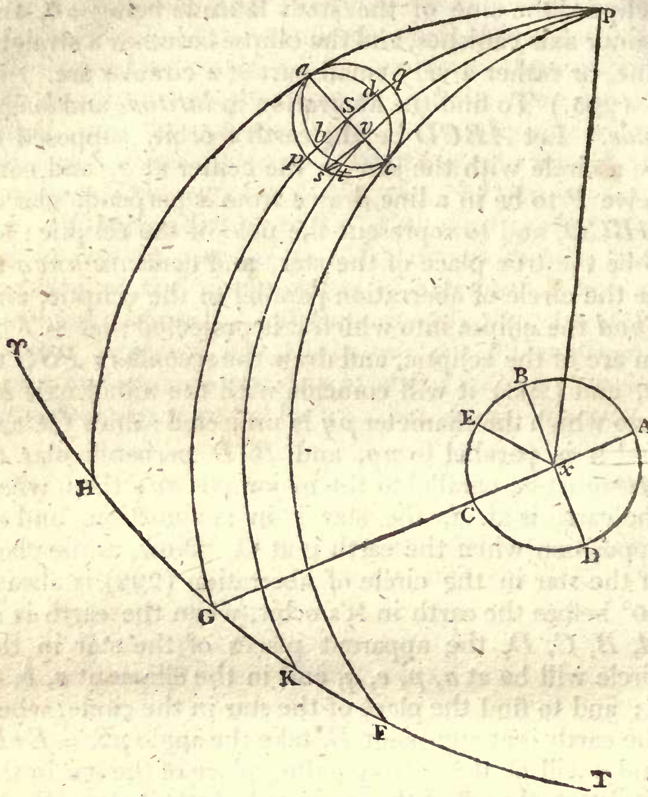
(292.) As we may, for the purposes which we shall here want to consider, conceive the earth's orbit $AFBQ$ to be a circle, and therefore to coincide with $AyZB$, if from the center C we draw Cs' parallel to Ss, or yF, s' will be the point in that circle corresponding to s in the circle $WsVx$; and as $Fs' = 90°$, the apparent place of the star in the circle of aberration is always 90° *before* the place of the earth in it's orbit, and consequently the apparent angular velocity of the star and earth about their respective centers are always equal. It is further supposed, that the star S' is at an indefinitely great distance; for the true place of the star is supposed not to be altered from the motion of the earth, and considering FS as always parallel to itself, it will always be directed to S' as a fixed point in the heavens. Hence also, as the apparent place of the sun is opposite to that of the earth, the apparent place of the star, in the circle of aberration, is 90° *behind* that of the sun.

(293.) As a small part of the heavens may be conceived as a plane perpendicular to a line joining the star and eye, it follows, from the principles of orthographic projection, that the circle *ambn* parallel to the ecliptic described by the apparent place of the star, projected upon that plane, will be an ellipse; the apparent path of the star in the heavens will therefore be an ellipse, and the major axis will be to the minor, as radius to the sine of the star's latitude. For let *CE* be the plane of the ecliptic, *P* it's pole, *PE* a secondary to it, *PC* perpendicular to *EC*, *C* the place of the eye, and let *ab* be parallel to *CE*, then it will be that diameter of the circle *anbm* of aberration which is seen most obliquely, and consequently that diameter which is projected into the minor axis of the ellipse; let *mn* be perpendicular to *ab*, and it will be seen directly, being perpendicular to a line drawn from it to the eye, and therefore will be the major axis; draw *Ca*, *Cbd*, and *ad* is the projection of *ab*; and as *ad* may be considered as a straight line, we have (Trig. Art. 128) *mn*, or *ab*, the major axis, : *ad* the minor :: rad. : sin. *abd*, or *ECd* the star's latitude. As the angle *bad* is the complement of *abd*, or of the star's latitude, the circle is projected upon a plane making an angle with it equal to the complement of the star's latitude.



(294.) As the minor axis *da* coincides with a secondary to the ecliptic, it must be perpendicular to it, and the major axis *mn* is parallel to it, it's position not being altered by projection. Hence, in the pole of the ecliptic, the sine of the star's latitude being radius, the ellipse becomes a circle; and in the plane of the ecliptic, the sine of the star's latitude being $=0$, the minor axis vanishes, and the ellipse becomes a straight line, or rather a very small part of a circular arc.

(295.) To find the aberration in *latitude* and *longitude*. Let $ABCD$ be the earth's orbit, supposed to be a circle with the sun in the center at x, and conceive P to be in a line drawn from x perpendicular to $ABCD$, and to represent the pole of the ecliptic; let S be the true place of the star, and conceive $apcq$ to be the circle of aberration parallel to the ecliptic, and $abcd$ the ellipse into which it is projected; let ♈T be an arc of the ecliptic, and draw the secondary PSG to it, and (293) it will coincide with the minor axis bd into which the diameter pq is projected; draw $GCxA$, and it is parallel to pq, and BxD perpendicular to AC must be parallel to the major axis ac; then, when the earth is at A, the star is in conjunction, and in opposition when the earth is at C. Now, as the place of the star in the circle of aberration (292) is always 90° before the earth in it's orbit, when the earth is at A, B, C, D, the apparent places of the star in the circle will be at a, p, c, q, and in the ellipse at a, b, c, d; and to find the place of the star in the circle, when the earth is at any point E, take the angle $pSs = ExB$, and s will be the corresponding place of the star in the circle; and to find the projected place in the ellipse, draw sv perpendicular to Sc, and vt perpendicular to Sc in the plane of the ellipse, and t will be the apparent place of the star in the ellipse; join st, and it will be perpendicular to vt, because the projection of the circle into the ellipse is in lines perpendicular to the ellipse; draw the secondary $PvtK$, which will, as to sense, coincide with vt, unless the star be very near to the pole of the ecliptic; therefore the rules here given will be sufficiently accurate, except in that case. Now as cvS is parallel to the ecliptic, S and v must have the same latitude; hence, vt is the aberration in latitude; and as G is the true, and K the apparent place of the star in the ecliptic, GK is the aberration



in longitude. To find these quantities, put m and n for the sine and cosine of the angle sSc, or CxE, the earth's distance from syzygies, radius being unity; and as (293) the angle $svt =$ the complement of the star's latitude, the angle $vst =$ the star's latitude, for the sine and cosine of which put v and w, and put $r = Sa$, or Ss; then in the right-angled triangle Ssv (Trig. Art. 128) $1 : m :: r : sv = rm$; hence, in the triangle vts, $1 : v :: rm : tv = rvm$ the aberration in *latitude*. Also, in the triangle Ssv, $1 : n :: r : vS = rn$; hence, w (13) $: 1 :: rn : GK = \frac{rn}{w}$ the aberration in *longitude*.

When the earth is in syzygies, $m=0$, therefore there is no aberration in latitude; and, as n is then greatest, there is the greatest aberration in longitude; if the earth be at *A*, or the star in conjunction, the apparent place of the star is at *a*, and reduced to the ecliptic at *H*; therefore *GH* is the aberration, which diminishes the longitude of the star, the order of the signs being ♈ *GT*; but when the earth is at *C*, or the star in opposition, the apparent place *c* reduced to the ecliptic is at *F*, and the aberration *GF* increases the longitude; hence, the longitude is the greatest when the star is in opposition, and least when in conjunction. When the earth is in quadratures at *D* or *B*, then $n=0$, and m is greatest; therefore there is no aberration in longitude, but the greatest in latitude; when the earth is at *D*, the apparent place of the star is at *d*, and the latitude is there increased; but when the earth is at *B*, the apparent place of the star is at *b*, and the latitude is diminished; hence, the latitude is least in quadrature before opposition, and greatest in quadrature after. From the mean of a great number of observations, Dr. *Bradley* determined the value of r to be $20''$.

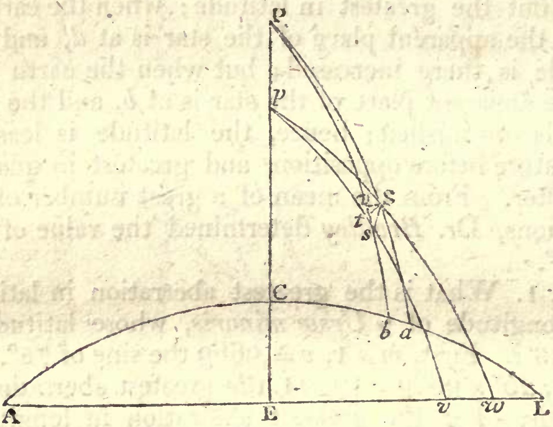
Ex. 1. What is the greatest aberration in latitude and longitude of γ *Ursæ minoris*, whose latitude is $75°.\ 13'$? First, $m=1$, $v=,9669$ the sine of $75°.\ 13'$; hence, $20''\times 9669=19,''34$ the greatest aberration in *latitude*. For the greatest aberration in longitude, $n=1$, $w=,2551$; hence, $\frac{20''}{,2551}=78'',4$ the greatest aberration in *longitude*.

Ex. 2. What is the aberration in latitude and longitude of the same star, when the earth is $30°$ from syzygies? Here $m=,5$; hence, $19'',34\times ,5=9'',67$ the aberration in *latitude*. If the earth be $30°$ beyond conjunction or before opposition, the latitude is diminished; but if it be $30°$ after opposition or before conjunction, the latitude is increased. Also, $n=,866$; hence, $78'',4\times ,866=67'',89$ the aberration in *longi-*

tude. If the earth be 30° from conjunction, the longitude is diminished; but if it be 30° from opposition, it is increased.

Ex. 3. For the *Sun*, $m=0$ and $n=1$, $w=1$; hence, it has no aberration in latitude, and the aberration in longitude $=r=20''$ constantly. This quantity $20''$ of aberration of the sun, answers to it's mean motion in $8'.\ 7''.\ 30'''$, which is therefore the time in which the light moves from the sun to the earth at it's mean distance. Hence, the sun always appears $20''$ backwarder than it's true place.

(296.) To find the aberration in *right ascension* and *declination*. Let AEL be the equator, p it's pole;

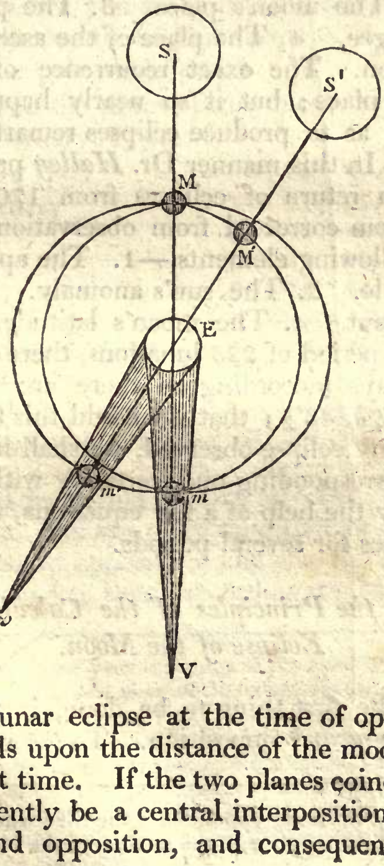


ACL the ecliptic, P it's pole; S the true place of the star, s the apparent place in the ellipse; draw the great circles, Psa, Psb, pSw, pSv, and Sv, st perpendicular to Pb, pv. Now $sv=rvm$ (295); also, $Sv=rn$; hence, (Trig. Art. 123) $rvm\ (vs) : rn\ (Sv) :: \text{rad.} :$ $\text{tan. } Ssv=\dfrac{n}{vm}=\dfrac{\text{cotan. earth's dist. syzy.}}{\text{sin. star's lat.}}=\text{cosec. star's lat.} \times \text{cotan. earth's distance from syzygies}$. Thus we immediately compute the angle Ssv; compute also the angle of position Psp from the three sides of that triangle being given (Trig. Art. 239), and we get the angle Ssp, it being the sum or difference of Ssv and Psp. Put a and b for the sine and cosine of Ssv, c and d for the sine and cosine of Ssp, $z =$ cosine of the star's declination; then (as sv, st, are the cosines of Ssv, Sst, to radius sS) $b : d :: sv\ (= rvm) : st = rvm \times \frac{d}{b} = 20'' \times vm \times \frac{d}{b}$ the aberration in *declination*; and (as Sv, St, are the sines of Ssv, Sst, to radius sS) $a : c :: Sv\ (= rn) : St = \frac{rnc}{a}$; hence (13), $vw \left(= \frac{St}{\text{cos. dec.}} \right) = 20'' \times \frac{nc}{az}$ the aberration in *right ascension*.

CHAP. XXI.

ON THE ECLIPSES OF THE SUN AND MOON.

(297.) AN eclipse of the *Moon* is caused by it's entering into the earth's shadow, and consequently it must happen when the moon is in opposition to the sun, or at the full moon. An eclipse of the *Sun* is caused by the interposition of the moon between the earth and sun, and therefore it must happen when the moon is in conjunction with the sun, or at the new moon. If the plane of the moon's orbit coincided with the plane of the ecliptic, there would be an eclipse at every opposition and conjunction; but the plane of the moon's orbit being inclined to the ecliptic, there can be no eclipse at opposition or conjunction, unless at that time the moon be at, or near to the node. For suppose *MM′mm′* be the orbit of the moon, and let the other circle represent the plane of the earth's orbit, or that plane in which the sun *S*, appears as seen from the earth *E*, and let these two planes be inclined to each other, so that we may conceive the part *MM′m* to lie above, and the part *mm′M* below the plane of the earth's orbit; and *M*, *m*, are the nodes. Now if the moon be at *M*, in conjunction, the three bodies are then in the same plane, and therefore the moon is interposed between the earth and sun, and causes an eclipse of the sun. But if the moon be at *M′* when the sun comes into conjunction at *S′*, *M′* is now elevated above the line joining *E* and *S′*, and *M′* may be so far from *M*, that the elevation of *M′* above the line *ES′* may be so much, that the moon may not be interposed between *E* and *S′*, in which case there will be no eclipse of the sun. Whether, therefore, there will be an eclipse of the sun at the conjunction, or not, depends upon the distance of the moon from the node at that time. If the moon be at *m* at the time of opposition, then the three bodies being in the same right line, the shadow *EV* of the earth must fall upon the moon, and the moon must suffer an eclipse. But if the moon be at *m* at the time of opposition, *m′* may be so far below the shadow *Ev* of the earth, that the moon may not pass through it, in which case there will be no eclipse. Whether, therefore, there



will be a lunar eclipse at the time of opposition, or not, depends upon the distance of the moon from the node at that time. If the two planes coincided, there would evidently be a central interposition every conjunction and opposition, and consequently a total eclipse. *Meton*, who lived about 430 years before CHRIST, observed, that after 19 years, the new and full moons returned again on the same day of the month. The ancient Astronomers also observed, that at the end of 18 years 10 days, a period of 223 lunations, there was a return of the same eclipses; and hence, they were enabled to foretel when they would happen. This is mentioned by *Pliny* the Naturalist, Lib. II. Ch. 13. and by *Ptolemy*, Lib. IV. Ch. 2. This restitution of eclipses depends upon the return of the following elements to the same state.—1. The sun's place. 2. The moon's place. 3. The place of the moon's apogee. 4. The place of the ascending node of the moon. The exact recurrence of these can never take place; but it so nearly happens in the above time, as to produce eclipses remarkably corresponding. In this manner Dr. *Halley* predicted and published a return of eclipses from 1700 to 1718, many of them corrected from observations; together with the following elements.—1. The apparent time of the middle. 2. The sun's anomaly. 3. The annual argument. 4. The moon's latitude. He says, that in this period of 223 lunations, there are 18 years 10 or 11 days (according as there are five or four leap-years) $7h.\ 43'\frac{3}{4}$; that if we add this time to the middle of any eclipse observed, we shall have the return of a corresponding one, certainly within $1h.\ 30'$; and that, by the help of a few equations, we may find the like series for several periods.

To explain the Principles of the Calculation of an Eclipse of the Moon.

(298.) The first thing to be done, is to find the time of the *mean** opposition. To get which, from the Tables of Epacts*, amongst the Tables of the moon's motion, take out the epact for the year and month, and subtract the sum from 29*d*. 12*h*. 44′. 3″, one synodic revolution of the moon, or two if necessary, so that the remainder may be less than a revolution, and that remainder gives the time of the *mean* conjunction. If to this we add 14*d*. 18*h*. 22′. 1″,5, half a revolution, it gives the time of the next *mean* opposition; or if we subtract, it gives the time of the preceding *mean* opposition. If it be leap-year, in January and February subtract a day from the sum of the epacts, before you make the subtraction. When the day of the *mean* conjunction is 0, it denotes the last day of the preceding month.

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* The time of the *mean* opposition is the time when the opposition would have taken place, if the motions of the bodies had been uniform.

Ex. To find the times of the *mean* new and full moons in February, 1795.

Epact 1795 - -	9^{d}.	11^{h}.	6′.	17″
February - - -	1.	11.	15.	57
	10.	22.	22.	14
	29.	12.	44.	3
Mean new moon -	18.	14.	21.	49
	14.	18.	22.	1,5
Mean full moon -	3.	19.	59.	47,5

(299.) To determine whether an eclipse may happen at opposition, find the *mean* longitude of the earth at the time of *mean* opposition, and also the longitude of the moon's node; then, according to M. *Cassini*, if the difference between the *mean* longitudes of the earth and the moon's node be less than 7°. 30′, there *must* be an eclipse; if it be greater than 14°. 30′, there *cannot* be an eclipse; but between 7°. 30′, and 14°. 30′, there *may*, or may *not*, be an eclipse. M. de *Lambre* makes these limits 7°. 47′, and 13°. 21′*.

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* The epact for any year is the age of the moon at the beginning of the year from the last *mean* conjunction, that is, from the time when the mean longitudes of the sun and moon were last equal. The epact for any month is the age which the moon would have had at the beginning of the month, if it's age had been nothing at the beginning of the year; therefore, if to the epact for the year, the epact for the month be added, the sum taken from 29*d*. 12*h*. 44′. 3″, or from twice that quantity if the sum exceed it, must give the time of *mean* conjunction.

Ex. To find whether there will be an eclipse at the full moon on February 3, 1795.

Sun's mean long. at $3^d.19^h.59'.47'',6$	$10^s.13°.27'.20''8$
Mean long. of the earth - - - -	4. 13. 27. 20,8
Long. of the moon's node - - -	4. 8. 1. 48,5
Difference - - - - - - - - -	0. 5. 25. 32,3

Hence, there must be an eclipse.

Examine thus all the new and full moons for a month before and a month after the time at which the sun comes to the place of the nodes of the lunar orbit, and you will be sure not to miss any eclipses. Or, having the eclipses for the last 18 years, if you add to the times of the middle of these eclipses, 18*y*. 10*d*. 7*h*. $43'\frac{3}{4}$, or 18*y*. 11*d*. 7*h*. $43'\frac{3}{4}$, (297) it will give the times when you may expect the eclipses will return.

(300.) To the time of *mean* opposition, compute the true longitudes of the sun and moon, and the moon's true latitude; and find, from the Tables of their motions, the horary motions of the sun and moon in longitude, and the difference (d) of their horary motions is the relative horary motion of the moon in respect to the sun, or the motion with which the moon *approaches to*, or *recedes from*, the sun; find also the moon's horary motion in latitude; and suppose, at the time (t) of *mean* opposition, the moon is at the distance (m) from opposition; then, as we may suppose the moon to approach the sun, or recede from it, uniformly, $d : m :: 1$ hour : the time (w) between t and the opposition, which added to, or subtracted from, the time t, according as the moon is not yet got into opposition, or is beyond it, gives the time of the ecliptic opposition.

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* This may be found from Art. 306. by finding the true limit, and then applying the greatest difference of the true and mean places.

(301.) To find the place of the moon in opposition, let n be the moon's horary motion in longitude; then, 1 hour : $w :: n$: the increase of the moon's longitude in the time w, which applied to the moon's longitude at the time of the mean opposition, gives the true longitude of the moon at the time of the ecliptic opposition. The opposite point to that must be the true longitude of the sun. Find also the moon's true latitude at the time of opposition, by saying, 1 hour : w :: the horary motion in latitude : the motion in latitude in the time w, which applied to the moon's latitude at the time of the mean opposition, gives the true latitude at the time of the true opposition*. In like manner you may compute the true time of the ecliptic conjunction, and the places of the sun and moon for that time, when you calculate a solar eclipse.

(302.) With the sun's horary motion in longitude, and the moon's in longitude and latitude, find the inclination of the *relative* orbit, and the horary motion upon it. To do this, let LM be the horary motion of the moon in longitude, SM that of the sun; draw Ma perpendicular to LM, and equal to the moon's horary motion in latitude; take $Sb = Ma$, and parallel to it, and join La, Lb; then La is the moon's *true* orbit, and Lb it's *relative* orbit in respect to the sun.

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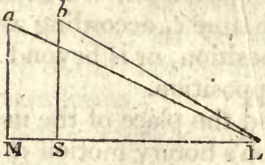
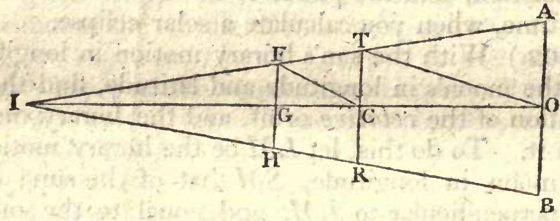
* For greater certainty, you may compute again, from the Tables, the places of the sun and moon, and if they be not exactly in opposition, which probably may not be the case, as the moon's longitude does not increase uniformly, repeat the operation. This accuracy, however, in eclipses is generally unnecessary; for the best lunar Tables cannot be depended upon to give the moon's longitude nearer than 30″; therefore the probable error from the Tables is vastly greater than that which arises from the motion in longitude not being uniform. Unless, therefore, very great accuracy be required, this operation is unnecessary.



Hence, LS (the difference of the horary motions in longitude) : Sb (the moon's horary motion in latitude :: radius : tan. bLS, the inclination of the relative orbit; and cos. bLS : radius :: LS : Lb, the horary motion in the *relative* orbit.

(303.) At the time of opposition, find, from the Tables, the moon's horizontal parallax, it's semidiameter, and the semidiameter of the sun, the horizontal parallax of which we may here take $=9''$.

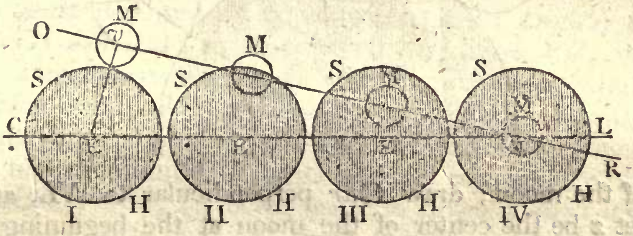
(304.) To find the semidiameter of the earth's shadow at the moon, seen from the earth. Let AB be the diameter of the sun, TR the diameter of the earth, O and C their centers; produce AT, BR, to meet at I, and draw OCI; let FGH be the diameter of the earth's shadow at the distance of the moon, and join OT, CF. Now the angle $FCG=CFA-CIA$, but $CIA=OTA-TOC$; therefore $FCG=CFA-OTA+TOC$, that is, *the angle under which the semidiameter of the earth's shadow, at the moon, appears, is equal to the sum of the horizontal parallaxes of the sun and moon diminished by the apparent semidiameter of the sun.* In eclipses of the moon, the shadow is found to be a little greater than this Rule



gives it, owing to the atmosphere of the earth. This augmentation of the semidiameter is, according to M. *Cassini*, 20″; according to M. *Monnier*, 30″; and according to M. de la *Hire*, 60″. *Mayer* thinks the correction is about $\frac{1}{60}$ of the semidiameter of the shadow, or that you may add as many seconds as the semidiameter contains minutes. Some computers always add 50″; but this must be subject to some uncertainty.

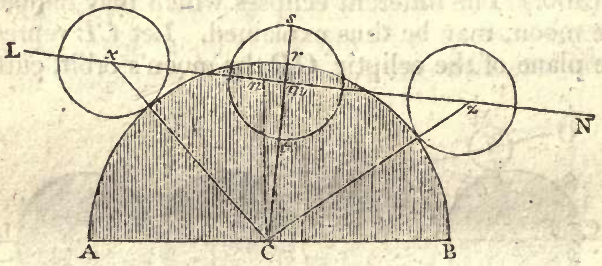
(305.) As the angle *CIT* $(=OTA-TOC)$ is known, we have sin. *TIC* : cos. *TIC* :: *TC* : *CI* the length of the earth's shadow. If we take the angle $ATO=16'.\ 3''$ the mean semidiameter of the sun, $TOC=9''$ the horizontal parallax of the sun, we have $CIT=15'.\ 54''$; hence, sin. $15'.\ 54''$: cos. $15'.\ 54''$, or $1 : 216{,}2 :: TC : CI=216{,}2\ TC$.

(306.) The different eclipses which may happen of the moon, may be thus explained. Let *CL* represent the plane of the ecliptic, *OR* the moon's orbit, cutting the ecliptic in the node *N*; and let *SH* represent a section of the earth's shadow at the distance of the moon from the earth, and *M* the moon at the time when she passes nearest to the center of the earth's shadow. Hence, if the opposition happen as in position I, it is manifest that the moon will just pass by the shadow of the earth without entering it, and there will be no eclipse. In position II, part of the moon will pass through the shadow, and there will be a *partial* eclipse. In position III, the whole of the



moon passes through the earth's shadow, and there is a *total* eclipse. In position IV, the center of the moon passes through the center of the earth's shadow, and there is a *total* and *central* eclipse. It is plain, therefore, that whether there will, or will not, be an eclipse at the time of opposition, depends upon the distance of the moon from the node at that time; or the distance of the earth's shadow, or of the earth, from the node. Now in lunar eclipses we may take the angle at $N = 5°. 17'$, and in position I. the value of Ev is about $1°. 3'. 30''$; hence (Trig. Art. 221), sin. $5°. 17'$: rad. :: sin. $1°. 3'. 30''$: sin. $QN = 11°. 34'$; when, therefore, EN is greater than that quantity at the time of opposition, there can be no eclipse. This quantity $11°. 34'$ is called the *ecliptic limit*.

(307.) Let ArB be that half of the earth's shadow which the moon passes through, NL the relative orbit of the moon; draw Cmr perpendicular to NL, and let z be the center of the moon at the beginning of the eclipse, m at the middle, x at the end; also, let AB be the ecliptic, and Cn perpendicular to it. Now in the right-angled triangle Cnm, we know Cn the latitude of the moon at the time of the ecliptic conjunction, and (302) the angle Cnm* the complement of the angle which the relative orbit of the moon makes with the ecliptic; hence (Trig. Art. 125), rad. : cos.



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[*] If the moon at n have north or south latitude increasing, the angle Cnm is to be set off to the right; otherwise, to the left of Cn.

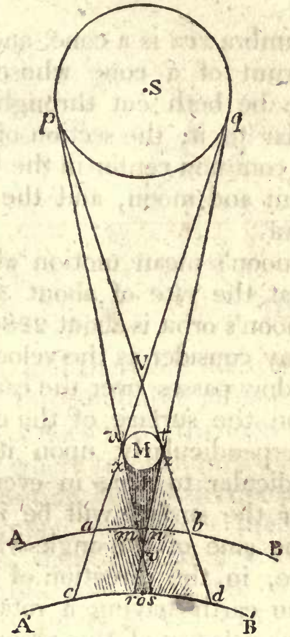
Cnm :: *Cn* : *nm*, which is called the *Reduction*; and rad. : sin. *Cnm* :: *Cn* : *Cm*. The horary motion (h) of the moon upon it's relative orbit being known, we know the time of describing *mn*, by saying, $h : mn ::$ 1 *hour* : *the time of describing mn.* Hence, knowing the time of the ecliptic conjunction at *n*, we know the time of the middle of the eclipse at *m*. Next, in the right-angled triangle *Cmz*, we know *Cm*, and *Cz* the sum of the semidiameters of the earth's shadow and the moon, to find *mz*, which is done thus by logarithms; as $mz = \sqrt{Cz^2 - Cm^2} = \sqrt{\overline{Cz + Cm} \times \overline{Cz - Cm}}$, the *log. of* $mz = \frac{1}{2} \times$ *log.* $\overline{Cz + Cm}$ + *log.* $\overline{Cz - Cm}$, (Trig. Art. 52). Hence, the horary motion of the moon being known, we know the time of describing *zm*, which *subtracted* from the time at *m* gives the time of the beginning, and *added*, gives the time of the end. The magnitude of the eclipse at the middle is represented by *tr*, which is the greatest distance of the moon within the earth's shadow, and this is measured in terms of the diameter of the moon, conceived to be divided into 12 equal parts, called *Digits*, or ***Parts deficient***; to find which, we know *Cm*, the difference between which and *Cr* gives *mr*, which added to *mt*, or if *m* fall *out* of the shadow, take the difference between *mr* and *mt*, and we get *tr*; hence, to find the number of digits eclipsed, say, *mt* : *tr* :: 6 *digits, or* 360′, (it being usual to divide a digit into 60 equal parts, and call them minutes,) : *the digits eclipsed.* If the latitude of the moon be north, we use the *upper* semicircle; if south, we take the *lower*.

(308.) If the earth had no atmosphere, when the moon was totally eclipsed, it would be invisible; but, by the refraction of the atmosphere, some rays will be brought to fall on the moon's surface, upon which account the moon will be visible at that time, and appear of a dusky red colour. M. *Maraldi* (*Mem. de l'Acad.* 1723) has observed, that, in general, the earth's umbra, at a certain distance, is divided by a kind of penumbra, from the refraction of the atmosphere. This will account for the circumstance of the moon being more visible in some total eclipses than in others. It is said, that the moon, in the total eclipses in 1601, 1620, and 1642, entirely disappeared.

(309.) An eclipse of the moon, arising from it's real deprivation of light, must appear to begin at the same instant of time to every place on that hemisphere of the earth which is turned towards the moon. Hence, it affords a very ready method of finding the difference of longitudes of places upon the earth, as will be afterwards explained. The moon enters the penumbra of the earth before it comes to the umbra, and therefore it gradually loses it's light; and the penumbra is so dark just at the umbra, that it is difficult to ascertain the exact time when the moon's limb touches the umbra, or when the eclipse begins. When the moon has entered into the umbra, the shadow upon it's disc is tolerably well defined, and you may determine, to a considerable degree of accuracy, the time when any spot enters into the umbra. Hence, the beginning and end of a lunar eclipse are not so proper to determine the longitude from, as the times at which the umbra touches any of the spots.

On Eclipses of the Sun.

(310.) An eclipse of the sun is caused by the interposition of the moon between the sun and spectator, or by the shadow of the moon falling on the earth at the place of the observer. The different kinds of eclipses will be best explained by a figure. Let S be the sun, M the moon, AB or $A'B'$ the surface of the earth; draw tangents $pxvs$, $qzvr$, from the sun to the *same* side of the moon, and xvz will be the moon's *umbra,* in which no part of the sun can be seen; if tangents $ptbd$, $qwac$, be drawn from the sun to the *opposite* sides of the moon, the space comprehended between the *umbra* and wac, tbd, is called the *penumbra,* in which part of the sun only is seen. Now it is manifest, that if *AB* be the surface of the

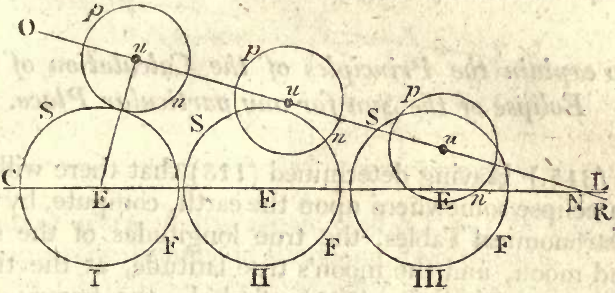


earth, the space *mn,* where the umbra falls, will suffer a *total* eclipse; the part *am, bn,* between the boundaries of the umbra and penumbra, will suffer a *partial* eclipse; but to all the other parts of the earth there will be no eclipse. Now let *A′B′* be the surface of the earth, the earth being, at different times, at different distances from the moon; then the space within *rs* will suffer an *annular* eclipse; for if tangents be drawn from any point *o* within *rs* to the moon, they must evidently fall within the sun, therefore the sun will appear all round about the moon in the form of a ring; the parts *cr, sd,* will suffer a *partial* eclipse; and the other parts of the earth will suffer no eclipse. In this case, there can be no total eclipse any where, as the moon's umbra does not reach the earth. According to M. du *Sejour*, an eclipse can never be annular longer than 12′. 24″, nor total longer than 7′. 58″.

(311.) The umbra *xvz* is a cone, and the penumbra *wcdt* the frustrum of a cone whose vertex is *V*. Hence, if these be both cut through their common axis perpendicular to it, the section of each will be a circle, having a common center in the line joining the centers of the sun and moon, and the penumbra includes the umbra.

(312.) The moon's mean motion about the center of the earth is at the rate of about 33′ in an hour; but 33′ of the moon's orbit is about 2280 miles, which, therefore, we may consider as the velocity with which the moon's shadow passes over the earth; but this is the velocity upon the surface of the earth where the shadow falls perpendicularly upon it, it being the velocity perpendicular to *Mv*; in every other place, the velocity over the surface will be increased in the proportion of the sine of the angle which *Mv* makes with the surface, in the direction of it's motion, to radius. But the earth having a rotation about it's axis, the *relative* velocity of the moon's shadow over any given point of the surface will be different from this; if the point be moving in the direction of the shadow, the velocity of the shadow, in respect to that point, will be diminished, and consequently the time in which the shadow passes over it will be increased; but if the point be moving in a direction contrary to that of the shadow, as is the case when the shadow falls on the other side of the pole, the time will be diminished. The length of a solar eclipse is therefore affected by the earth's rotation about it's axis.

(313.) The different eclipses of the sun may be thus explained. Let *CL* represent the orbit of the earth, *OR* the line described by the centers of the moon's umbra and penumbra at the earth; *N* the moon's node; *SF* the earth, *E* it's center; *pn* the moon's penumbra, u the umbra. Then, in position I, the penumbra pn just passes by the earth, without



falling upon it, and therefore there will be no eclipse. In position II, the penumbra pn falls upon the earth, but the umbra u does not; therefore there will be a *partial* eclipse where the penumbra falls, but no total eclipse. In position III, both the penumbra pn and umbra u fall upon the earth; therefore, where the penumbra falls, there will be a *partial* eclipse, and where the umbra falls there will be a *total* eclipse; and to the other parts of the earth there will be no eclipse. Now the *ecliptic limit*, may be thus found. The angle N may be taken 5°. 17′, and in position I, the value of Eu (u being the center of the umbra) is about 1°. 34′. 27″; hence (Trig. Art. 221) sin. 5°. 17′ : rad. :: sin. 1°. 34′. 27″ : sin. $EN = 17°. 21′. 27″$ the ecliptic limit; if therefore, at the time of conjunction, the earth be within this distance of the node, there will be an eclipse.

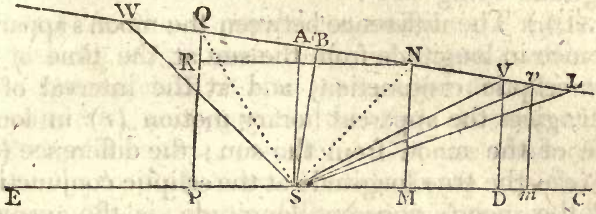
(314.) An eclipse of the sun, or rather of the earth, without respect to any particular place, may be calculated exactly in the same manner as an eclipse of the moon, that is, the times when the moon's umbra or penumbra first touches and leaves the earth; but to find the times of the beginning, middle, and end, at any particular place, the *apparent* place of the moon, as seen from thence, must be determined, and consequently it's parallax in latitude and longitude must be computed, which renders the calculation of a solar eclipse extremely long and tedious.

To explain the Principles of the Calculation of an Eclipse of the Sun for any particular Place.

(315.) Having determined (113) that there will be an eclipse somewhere upon the earth, compute, by the Astronomical Tables, the true longitudes of the sun and moon, and the moon's true latitude, at the time of mean conjunction (301); find also the horary motions of the sun and moon in longitude, and the moon's horary motion in latitude; and compute the time of the ecliptic conjunction of the sun and moon, in the same manner (300) as the time of the ecliptic opposition was computed. At the time of the ecliptic conjunction, compute (301) the sun's and moon's longitude, and the moon's latitude; find also the horizontal parallax of the moon from the Tables of the moon's motion, from which subtract the sun's horizontal parallax, and you get the horizontal parallax of the moon from the sun.

(316.) To the latitude of the given place, and the horizontal parallax of the moon from the sun (which we here use instead of the horizontal parallax of the moon, as we want to find what effect the parallax has in altering their apparent relative situations,) at the time of the ecliptic conjunction, compute (144) the moon's parallax in latitude and longitude from the sun; the parallax in latitude applied to the true latitude gives the apparent latitude (*L*) of the moon from the sun; and the parallax in longitude shows the apparent difference (*D*) of the longitudes of the sun and moon.

(317.) Let *S* be the sun, *CE* the ecliptic, according to the order of the signs; take $SM = D$, draw MN perpendicular to MS, and take it $= L$, then N is the



apparent place of the moon, and $SN = \sqrt{D^2 + L^2}$ is the apparent distance of the moon from the sun.

(318.) If the moon be to the *east* of the nonagesimal degree, the parallax *increases* the longitude; if to the *west*, it *diminishes* it (Art. 144); hence, if the *true* longitudes of the sun and moon be equal, in the former case the apparent place will be from S towards E, and in the latter, towards C. To some time, as an hour, *after* the true conjunction, if the moon be to the *west* of the nonagesimal degree; or *before* the true conjunction, if the moon be to the *east* of the nonagesimal degree, find the sun's and the moon's true longitude, and the moon's true latitude, from their horary motions; and to the same time compute the moon's parallax in latitude and longitude from the sun; apply the parallax in latitude to the true latitude, and it gives the apparent latitude (l) of the moon from the sun; take the difference of the sun's and moon's true longitude, and apply the parallax in longitude, and it gives the apparent distance (d) of the moon from the sun in longitude. From S set off $SP = d$, and on EC erect the perpendicular PQ equal to l, and Q is the apparent place of the moon at one hour from the true conjunction; and SQ $(= \sqrt{d^2 + l^2})$ is the apparent distance of the moon from the sun; draw the straight line NQ, and it will represent the relative *apparent* path of the moon, considered as a straight line, in general it being very nearly so; it's value also represents the relative horary motion of the moon in the apparent orbit, the relative horary motion in longitude being MP.

(319.) The difference between the moon's apparent distance in longitude from the sun at the time of the true ecliptic conjunction, and at the interval of an hour, gives the apparent horary motion (r) in longitude of the moon from the sun; the difference (D) between the true longitude at the ecliptic conjunction, and the moon's apparent longitude, is the apparent distance of the moon from the sun in longitude at the true time of the ecliptic conjunction; hence, $r : D :: 1$ hour : the time from the true to the apparent conjunction, consequently we know the time of the apparent conjunction. To find whether this time is accurate, we may compute (from the horary motions of the sun and moon) their true longitudes, and the moon's parallax in longitude from the sun, and apply it to the true longitude, and it gives the apparent longitude, and if this be the same as the sun's longitude, the time of the apparent conjunction is truly found; if they be not the same, find from thence the true time, as before. To the true time of the apparent conjunction, find the moon's true latitude from it's horary motion, and compute the parallax in latitude, and you get the apparent latitude at the time of the apparent conjunction. Draw SA perpendicular to CE, and equal to this apparent latitude; then the point A will not probably fall in NQ; but suppose it to fall in QN, to which draw SB perpendicular, and NR parallel to PM. Then knowing NR ($=PM$), and QR ($=QP \sim MN$) we have

$NR : RQ ::$ rad. : tan. QNR, or ASB (Trig. Art. 123)
Sin. QNR : rad. $:: QR : QN$ (Trig. Art. 128)

The time of describing NQ in the apparent orbit being equal to the time from M to P in longitude, NQ is the horary motion in the apparent orbit.

Rad. : sin. ASB :: AS : AB (Trig. Art. 125)
Rad. : cos. ASB :: AS : SB.

(320.) At the apparent conjunction the moon appears at A, which time (319) is known; when the moon appears at B, it is at it's nearest distance from the sun, and consequently the time is that of the greatest obscuration, (usually called the time of the middle,) provided there is an eclipse, which will always be the case, when SB is less than the *sum* of the apparent semidiameters of the sun and moon. If, therefore, it appear that there will be an eclipse, we proceed thus to find it's quantity, and the beginning and end. As we may suppose the motion to be uniform, QN : AB :: the time of describing NQ : the time of describing AB, which added to or subtracted from the time at A, (according as the apparent latitude is decreasing or increasing), gives the time of the greatest obscuration.

(321.) From the sum of the apparent semidiameters of the sun and moon subtract BS, and the remainder shows how much of the sun is covered by the moon, or the parts deficient; hence, semid. ⊙ : parts deficient :: 6 digits : the digits eclipsed. If SB be less than the *difference* of the semidiameters of the sun and moon, and the moon's semidiameter be the *greater*, the eclipse will be *total*; but if it be the *less*, the eclipse will be *annular*, the sun appearing all round the moon; if B and S coincide, the eclipse will be *central*.

(322.) Produce, if necessary, QN, and take SV, SW, equal to the sum of the apparent semidiameters of the sun and moon, at the beginning and end respectively; then $BV = \sqrt{SV^2 - SB^2}$, and $BW = \sqrt{SW^2 - SB^2}$; and to find the times of describing these, say, as the hourly motion of the moon in the apparent orbit, or NQ, : BV :: 1 hour : the time of describing VB; and NQ : BW :: 1 hour : the time of describing BW, which times, respectively subtracted from and added to the time of the greatest obscuration, give nearly the times of the beginning and end. But if accuracy be required, a different method must be adopted; for we suppose VW to be a straight line, which supposition will, in general, cause errors, too considerable to be neglected. It may, however, always serve as a rule to assume the time of the beginning and end. Hence it follows, that the time of the greatest obscuration at B, is not necessarily equidistant from the beginning and end.

(323.) If the eclipse be total, take SV, SW, equal to the difference of the semidiameters of the sun and moon, and then $BV = BW = \sqrt{SW^2 - SB^2}$, from whence we may find the times of describing BV, BW, as before, which we may consider as equal, and which applied to the time of the greatest obscuration at B, give the time of the beginning and end of the total darkness.

(324.) To find more accurately the time of the beginning and end of the eclipse, we must proceed thus. At the estimated time of the beginning, find, from the horary motions, and the computed parallaxes, the apparent latitude VD of the moon, and it's apparent longitude DS from the sun, and we have $SV = \sqrt{SD^2 + DV^2}$, and if this be equal to the apparent semid. ☽ + semid. ☉ (which sum call S), the estimated time is the time of the beginning; but if SV be not equal to S, assume (as the error directs) another time at a small interval from it, *before*, if SV be *less* than S, but *after*, if it be *greater*; to that time compute again the moon's apparent latitude mv, and apparent longitude Sm from the sun, and find $Sv = \sqrt{Sm^2 + mv^2}$; and if this be not equal to S, proceed thus; as the difference of Sv and SV : the difference of Sv and SL $(= S)$ :: the above-assumed interval of time, or time of the motion through Vv, : the time through vL, which added to or subtracted from the time at v, according as Sv is greater or less than SL, gives the time of the beginning. The reason of this operation is, that as Vv, vL, are very small, they will be very nearly proportional to the differences of SV, Sv, and Sv, SL. But as the variation of the apparent distance of the sun from the moon is not exactly in proportion to the variation of the differences of the apparent longitudes and latitudes, in cases where the utmost accuracy is required, the time of the beginning thus found (if it appear to be not correct) may be corrected, by assuming it for a third time, and proceeding as before. This correction, however, will never be necessary, except where extreme accuracy is required in order to deduce some consequences from it. But the time thus found is to be considered as accurate, only so far as the Tables of the sun and moon can be depended upon for their accuracy; and the best lunar Tables are subject to an error of 30″ in latitude. Hence, accurate observations of an eclipse, compared with the computed time, furnish the means of correcting the lunar Tables. In the same manner, the end of the eclipse may be computed.

(325.) As there are not many persons who have an opportunity of seeing a total eclipse of the sun, we shall here give the phænomena which attended that on April 22, 1715. Capt. *Stannyan*, at Bern in Switzerland, says, “ the sun was totally dark for four minutes and a half; that a fixed star and planet appeared very bright; and that it's getting out of the eclipse was preceded by a blood-red streak of light, from it's left limb, which continued not longer than six or seven seconds of time; then part of the sun's disc appeared, all on a sudden, as bright as *Venus* was ever seen in the night; nay, brighter, and in that very instant gave a light and shadow to things, as strong as moon-light used to do.” The inference drawn from these phænomena is, that the moon has an atmosphere.

J. C. Facis, at Geneva, says, “ there was seen, during the whole time of the total immersion, a whiteness, which seemed to break out from behind the moon, and to encompass it on all sides equally; it's breadth was not the twelfth part of the moon's diameter. *Venus*, *Saturn*, and *Mercury* were seen by many; and if the sky had been clear, many more stars might have been seen, and with them *Jupiter* and *Mars*. Some gentlewomen in the country saw more than 16 stars; and many people on the mountains saw the sky starry, in some places where it was not overcast, as during the night at the time of the full moon. The duration of the total darkness was three minutes."

Dr. *J. J. Scheuchzer*, at Zurich, says, "that both planets and fixed stars were seen; the birds went to roost; the bats came out of their holes; and the fishes swam about; we experienced a manifest sense of cold; and the dew fell upon the grass. The total darkness lasted four minutes."

(326.) Dr. *Halley**, who observed this eclipse at London, has thus given the phænomena attending it. "It was *universally* observed, that when the last part of the sun remained on it's east side, it grew very faint, and was easily supportable to the naked eye, even through the telescope, for above a minute of time before the total darkness; whereas, on the contrary, my eye could not endure the splendor of the emerging beams in the telescope from the first moment. To this, perhaps, two causes concurred; the one, that the pupil of the eye did necessarily dilate itself during the darkness, which before had been much contracted by looking on the sun. The other, that the eastern parts of the moon, having been heated with a day near as long as thirty of our's, must of necessity have that part of it's atmosphere replete with vapours, raised by the long-continued action of the sun; and, by consequence, it was more dense near the moon's surface, and more capable of obstructing the lustre of the sun's beams. Whereas at the same time the western edge of the moon had suffered as long a night, during which time there might fall in dews, all the vapours that were raised in the preceding long day; and for this reason, that part of it's atmosphere might be seen much more pure and transparent.

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* The Doctor begins his account thus. "Though it be certain, from the principles of Astronomy, that there happens necessarily a central eclipse of the sun, in some part or other of the terraqueous globe, about twenty-eight times in each period of eighteen years; and that of these, no less than eight do pass over the parallel of London, three of which eight are total with continuance; yet, from the great variety of the elements, whereof the *calculus* of eclipses consists, it has so happened, that since March 20, 1140, I cannot find that there has been a total eclipse of the sun seen at London, though in the mean time the shade of the moon has often passed over other parts of Great Britain."

About two minutes before the total immersion, the remaining part of the sun was reduced to a very fine horn, whose extremities seemed to lose their acuteness, and to become round like stars. And for the space of about a quarter of a minute, a small piece of the southern horn of the eclipse seemed to be cut off from the rest by a good interval, and appeared like an oblong star round at both ends; which appearance could proceed from no other cause, but the inequalities of the moon's surface, there being some elevated parts thereof near the moon's southern pole, by which interposition, part of that exceedingly fine filament of light was intercepted.

A few seconds before the sun was totally hid, there discovered itself round the moon a luminous ring, about a digit, or perhaps a tenth part, of the moon's diameter in breadth. It was of a pale whiteness, or rather pearl colour, seeming to me a little tinged with the colours of the *iris*, and to be concentric with the moon; whence I concluded it was the moon's atmosphere. But the great height of it, far exceeding that of our earth's atmosphere; and the observations of some one who found the breadth of the ring to increase on the west side of the moon, as the emersion approached; together with the contrary sentiments of those, whose judgement I shall always revere, make me less confident, especially as in a matter whereto I gave not all the attention requisite.

Whatever it was, this ring appeared much brighter and whiter near the body of the moon, than at a distance from it; and it's outward circumference, which was ill defined, seemed terminated only by the extreme rarity of the matter it was composed of; and in all respects resembled the appearance of an enlightened atmosphere viewed from far: but whether it belonged to the sun or the moon, I shall not at present undertake to decide.

During the whole time of the total eclipse, I kept my telescope constantly fixed on the moon, in order to observe what might occur in this uncommon appearance, and I saw perpetual flashes or coruscations of light, which seemed for a moment to dart out from behind the moon, now here, now there, on all sides, but more especially on the western side, a little before the emersion; and about two or three seconds before it, on the same western side, where the sun was just coming out, a long and very narrow streak of dusky, but strong red light, seemed to colour the dark edge of the moon, though nothing like it had been seen immediately after the immersion. But this instantly vanished upon the first appearance of the sun, as did also the aforesaid luminous ring.

As to the degree of darkness, it was such, that one might have expected to have seen more stars than were seen in London; the planets, *Jupiter*, *Mercury*, and *Venus*, were all that were seen by the gentlemen of the Society from the top of their house, where they had a free horizon; and I do not hear that any one in town saw more than *Capella* and *Aldebaran* of the fixed stars. Nor was the light of the ring round the moon capable of effacing the lustre of the stars, for it was vastly inferior to that of the full moon, and so weak, that I did not observe it cast a shade. But the under-parts of the hemisphere, particularly in the south-east under the sun, had a crepuscular brightness; and all round us, so much of the segment of our atmosphere as was above the horizon, and was without the cone of the moon's shadow, was more or less enlightened by the sun's beams; and it's reflection gave a diffused light, which made the air seem hazy, and hindered the appearance of the stars. And that this was the real cause thereof, is manifest by the darkness being more perfect in those places near which the center of the shade passed, where many more stars were seen, and in some, not less than twenty, though the light of the ring was to all alike.

I forbear to mention the chill and damp, with which the darkness of this eclipse was attended, of which most spectators were sensible, and equally judges; or the concern that appeared in all sorts of animals, birds, beasts, and fishes, upon the extinction of the sun, since ourselves could not behold it without some sense of horror."

(327.) If a conjunction of the sun and moon happen at, or very near, the node, there will be a great solar eclipse; but, in this case, at the preceding opposition, the earth was not got into the lunar ecliptic limits, and at the next opposition it will be got beyond it; hence, at each node there may happen only one solar eclipse, and therefore in a year there *may* happen only two solar eclipses.

There must be one conjunction in the time in which the earth passes through the solar ecliptic limits, and consequently there must be one solar eclipse at each node; hence, there *must* be two solar eclipses at least in a year.

If an opposition happen just before the earth gets into the lunar ecliptic limit, the next opposition may not happen till the earth is got beyond the limit on the other side of the node; consequently there may not be a lunar eclipse at the node; hence, there *may* *not* be an eclipse of the moon in the course of a year. When, therefore, there are only two eclipses in a year, they must be both of the sun.

If there be an eclipse of the moon as soon as the sun gets within the lunar ecliptic limit, it will be got out of the limit before the next opposition; consequently there can be only one lunar eclipse at the same node. But as the nodes of the moon's orbit move backwards about 19° in a year, the earth may come within the lunar ecliptic limits, at the *same* node, a second time in the course of a year, and therefore there *may* be three lunar eclipses in a year; and there can be no more.

If an eclipse of the moon happen at, or very near to, the node, a conjunction may happen before and after, whilst the earth is within the solar ecliptic limits; hence there may, at each node, happen two eclipses of the sun and one of the moon; and in this case, the eclipses of the sun will be small, and that of the moon large. When, therefore, the eclipses do not happen a second time at either node, there may be six eclipses in a year, four of which will be of the sun, and two of the moon. But if, as in the last case, an eclipse should happen at the return of the earth within the lunar ecliptic limits at the *same* node a second time in the year, there may be six eclipses, three of the sun and three of the moon.

There may be seven eclipses in a year. For twelve lunations are performed in 354 days, or in 11 days less than a common year. If, therefore, an eclipse of the sun should happen before January 11, and there be at that, and at the next node, two solar and one lunar eclipse at each; then the twelfth lunation from the first eclipse will give a new moon within the year, and (on account of the retrograde motion of the moon's nodes) the earth may be got within the solar ecliptic limits, and there may be another solar eclipse. Hence, when there are seven eclipses in a year, five will be of the sun and two of the moon. This is upon supposition that the first eclipse is of the sun; but if the first eclipse should be of the moon, there may be three of the sun and four of the moon.

As there are seven eclipses in the year but seldom, the mean number will be about four.

The nodes of the moon move backwards about 19° in a year, which arc the earth describes in about 19 days, consequently the middle of the seasons of the eclipses happens every year about 19 days sooner than in the preceding year.

The ecliptic limits of the sun (313) are greater than those of the moon (306), and hence, there will be more solar than lunar eclipses, in about the same proportion as the limit is greater, that is, as 3 : 2 nearly. But more lunar than solar eclipses are seen at any given place, because a lunar eclipse is visible to a whole hemisphere at once; whereas a solar eclipse is visible only to a part, and therefore there is a greater probability of seeing a lunar than a solar eclipse. Since the moon is as long above the horizon as below, every spectator may expect to see half the number of lunar eclipses which happen.

CHAP. XXII.

ON THE TRANSITS OF MERCURY AND VENUS OVER THE SUN'S DISC.

(328.) WHEN Dr. *Halley* was at St. Helena, whither he went for the purpose of making a catalogue of the stars in the southern hemisphere, he observed a transit of *Mercury* over the sun's disc; and, by means of a good telescope, it appeared to him that he could determine the time of the ingress and egress, without it's being subject to an error of 1″ *; upon which he immediately concluded, that the sun's parallax might be determined by such observations, from the difference of the times of the transit over the sun, at different places upon the earth's surface. But this difference is so small in Mercury, that it would render the conclusion subject to a great degree of inaccurary; in Venus, however, whose parallax is nearly four times as great as that of the sun, there will be a very considerable difference between the times of the transits seen from different parts of the earth, by which the accuracy of the conclusion will be proportionably increased. The Doctor, therefore, proposed to determine the sun's parallax from the transit of Venus over the sun's disc, observed at different places on the earth; and as it was not probable that he himself should live to observe the next transits, which happened in 1761 and 1769, he very earnestly recommended the attention of them to the Astronomers who should be alive at that time. Astronomers were therefore sent from England and France to the most proper parts of the earth to observe both those transits, from the result of which, the parallax has been determined to a very great degree of accuracy.

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* Hence, Dr. *Halley* concluded, that by a transit of *Venus*, the sun's distance might be determined with certainty to the 500th part of the whole; but the observations upon the transits which happened in 1761 and 1769, showed that the time of contact of the limbs of the Sun and Venus could not be determined to that degree of certainty.

(329.) *Kepler* was the first person who predicted the transits of Venus and Mercury over the sun's disc; he foretold the transit of Mercury in 1631, and the transits of Venus in 1631 and 1761. The first time *Venus* was ever seen upon the sun, was in the year 1639, on November 24, at Hoole, near Liverpool, by our countryman Mr. *Horrox,* who was educated at Emanuel College in this University. He was employed in calculating an Ephemeris from the *Lansberg* Tables, which gave, at the conjunction of Venus with the sun on that day, it's apparent latitude less than the semidiameter of the sun. But as these Tables had so often deceived him, he consulted the Tables constructed by *Kepler*, according to which, the conjunction would be at 8*h.* 1′ A. M. at Manchester, and the planet's latitude 14′. 10″ south; but, from his own corrections, he expected it to happen at 3*h.* 57′ P. M. with 10′ south latitude. He accordingly gave this information to his friend Mr. *Crabtree,* at Manchester, desiring him to observe it; and he himself also prepared to make observations upon it, by transmitting the sun's image through a telescope into a dark chamber. He described a circle of about six inches diameter, and divided the circumference into 360°, and the diameter into 120 equal parts, and caused the sun's image to fill up the circle. He began to observe on the 23d, and repeated his observations on the 24th till one o'clock, when he was unfortunately called away by business; but, returning at 15′ after three o'clock, he had the satisfaction of seeing Venus upon the sun's disc, just wholly entered on the left side, so that the limbs perfectly coincided. At 35′ after three, he found the distance of Venus from the sun's center to be 13′. 30″; and at 45′ after three, he found it to be 13′; and the sun setting at 50′ after three o'clock, put an end to his observations. From these observations, Mr. *Horrox* endeavoured to correct some of the elements of the orbit of Venus. He found Venus had entered upon the disc at about 62°. 30′ from the vertex towards the right on the image, which, by the telescope, was inverted. He measured the diameter of Venus, and found it to be to that of the sun, as 1,12 : 30, as near as he could measure. Mr. *Crabtree*, on account of the clouds, got only one sight of Venus, which was at 3*h*. 45′. Mr. *Horrox* * wrote a Treatise, entitled *Venus in Sole visa*, but did not live to publish it; it was, however, afterwards published by *Hevelius*. *Gassendus* observed the transit of *Mercury* which happened on November 7, 1631, and this was the first which had ever been observed; he made his observations in the same manner that *Horrox* did after him. Since his time, several transits of Mercury have been observed, as they frequently happen; whereas only two transits of Venus have happened since the time of *Horrox*. If we know the time of the transit at one node, we can determine, in the following manner, when they will probably happen again at the same node.

(330.) The mean time from conjunction to conjunction of Venus or Mercury being known (Art. 201), and the time of one mean conjunction, we shall know the time of all the future mean conjunctions; observe, therefore, those which happen near to the node, and compute the geocentric latitude of the planet at the time of conjunction, and if it be less than the apparent semidiameter of the sun, there will be a transit of the planet over the sun's disc; and we may determine the periods when such conjunctions happen, in the following manner. Let $P=$ the periodic time of the earth, p that of Venus or Mercury. Now that a transit may happen again at the same node, the earth must perform a certain number of complete revolutions in the same time that the planet performs a certain number, for then they must come into conjunction again at the same point of the earth's orbit, or nearly in the same position in respect to the node. Let the earth perform x revolutions whilst the planet performs y revolutions; then will $Px=py$, therefore $\frac{x}{y}=\frac{p}{P}$. Now $P=365{,}256$, and for *Mercury*, $p=87{,}968$; therefore $\frac{x}{y}=\frac{p}{P}=\frac{87{,}968}{365{,}256}=$ (by resolving it into it's continued fractions) $\frac{1}{4}, \frac{6}{25}, \frac{7}{29}, \frac{13}{54}, \frac{33}{137}, \frac{46}{191}$, &c. That is, 1, 6, 7, 13, 33, 46, &c. revolutions of the earth are nearly equal to 4, 25, 29, 54, 137, 191, &c. revolutions of Mercury, approaching nearer to a state of equality, the further you go. The first period, or that of one year, is not sufficiently exact; the period of six years will sometimes bring on a return of the transit at the *same* node; that of seven years more frequently; that of 13 years still more frequently, and so on. Now there was a transit of Mercury at it's descending node, in May, 1786; hence, by continually adding 6, 7, 13, 33, 46, &c. to it, you get all the years when a transit may be expected to happen at that node. In 1789, there was a transit at the ascending node, and therefore, by adding the same numbers to that year, you will get the years in which the transits may be expected to happen at that node. The next transits at the descending node will happen in 1832, 1845, 1878, 1891; and at the ascending node, in 1815, 1822, 1835, 1848, 1861, 1868, 1881, 1894.

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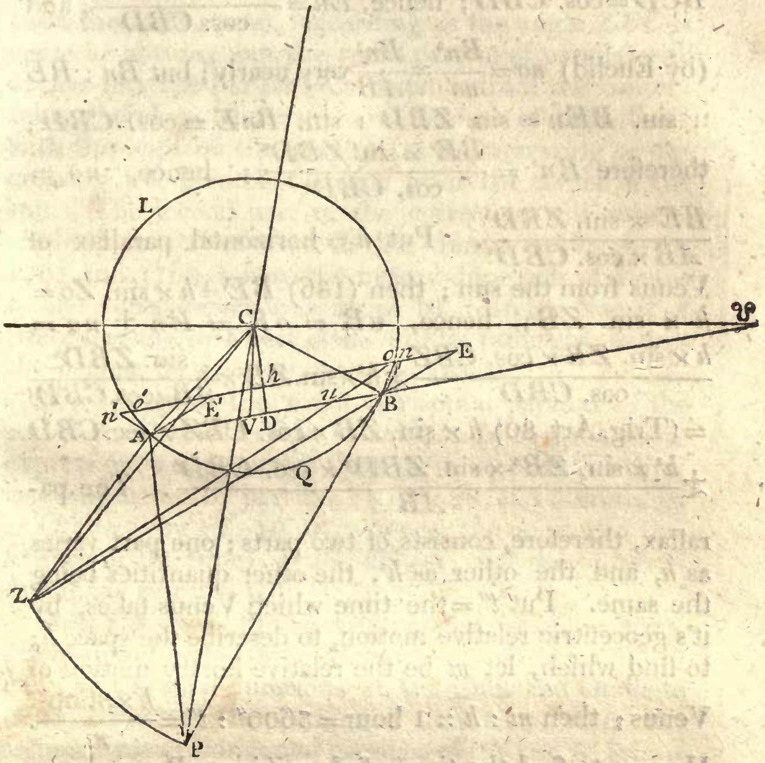
* The difficulties which this very extraordinary person had to encounter with in his astronomical pursuits, he himself has described, in the *Prolegomena* prefixed to his *Opera Posthuma*, published by Dr. *Wallis*.

For *Venus*, $p = 224{,}7$; hence, $\frac{x}{y} = \frac{p}{P} = \frac{224{,}7}{365{,}256} = \frac{8}{13}, \frac{235}{382}, \frac{713}{1159}$, &c. Therefore the periods are 8, 235, 713, &c. years. The transits at the same node will therefore, sometimes, return in eight years, but oftener in 235, and still oftener in 713, &c. Now, in 1769, a transit happened at the descending node in June, and the next transits at the same node will be in 2004, 2012, 2247, 2255, 2490, 2498, 2733, 2741, and 2984. In 1639, a transit happened at the ascending node in November; and the next transits at the same node will be in 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, 2846, and 2854. These transits are found to happen, by continually adding the periods, and finding the years when they may be expected, and then computing, for each time, the shortest geocentric distance of Venus from the sun's center at the time of conjunction, and if it be less than the semidiameter of the sun, there will be a transit.

A new Method of computing the Effect of Parallax, in accelerating or retarding the Time of the Beginning or End of a Transit of Venus *or* Mercury *over the Sun's Disc. By* NEVIL MASKELYNE, *D.D. F.R.S. and Astronomer Royal.*

(331.) The scheme which is here given, relates particularly to the transit of *Venus* over the sun which happened in 1769. Let *C* represent the center of the sun *LQ*, *P* the celestial north pole of the equator, *PC* a meridian passing through the sun, *Z* the zenith of the place, *ADB* ☋ the relative path of Venus, ☋ being the relative place of the descending node; *A* the geocentric place of Venus at the ingress, *B* at the egress, and *D* at the nearest approach to the sun's center, as seen from the earth's center, and *o* the apparent place of Venus at the egress to an observer whose zenith is Z; draw ouZ, and u is the *true* place of Venus when the *apparent* place is at o, and no is the



parallax in altitude of Venus from the sun; and the time of contact will be diminished by the time which Venus takes to describe uB; draw $n'o'honE$ parallel to AB, meeting ZB produced in E, and Bn, An', tangents to the circle, and let ChD be perpendicular to AB. Now the trapezium $uoEB$, on account of the smallness of it's sides, may be considered as rectilinear, and from the magnitude of ZB compared with Bu, BE may be considered as parallel to uo, consequently $uoEB$ may be considered as a parallelogram, and therefore Eo may be taken equal to Bu. Now $Eo=En\pm no$, according as E falls without or within the circle LQ of the sun's disc; and (Trig. Art. 128) $En : EB :: \text{sin. } EBn = \text{cos. } CBZ : \text{sin. } BnE = \text{sin. } BCD = \text{cos. } CBD$; hence, $En = \frac{EB \times \text{cos. } CBZ}{\text{cos. } CBD}$; and (by Euclid) $no = \frac{Bn^2}{no'} = \frac{Bn^2}{AB}$ very nearly; but $Bn : BE :: \text{sin. } BEn = \text{sin. } ZBD : \text{sin. } BnE = \text{cos. } CBD$; therefore $Bn^2 = \frac{BE^2 \times \text{sin. } ZBD^2}{\text{cos. } CBD^2}$; hence, $no = \frac{BE^2 \times \text{sin. } ZBD^2}{AB \times \text{cos. } CBD^2}$. Put h = horizontal parallax of Venus from the sun; then (136) $BE = h \times \text{sin. } Zo = h \times \text{sin. } ZB$; hence, $uB = oE = En \pm no = \frac{h \times \text{sin. } ZB \times \text{cos. } CBZ}{\text{cos. } CBD} \pm h^2 \times \text{sin. } ZB^2 \times \frac{\text{sin. } ZBD^2}{AB \times \text{cos. } CBD^2}$ = (Trig. Art. 80) $h \times \text{sin. } ZB \times \text{cos. } CBZ \times \text{sec. } CBD \pm \frac{h^2 \times \text{sin. } ZB^2 \times \text{sin. } ZBD^2 \times \text{sec. } CBD^2}{AB}$. The parallax, therefore, consists of two parts; one part varies as h, and the other as h^2, the other quantities being the same. Put t'' = the time which Venus takes, by it's geocentric relative motion, to describe the space h; to find which, let m be the relative horary motion of Venus; then $m : h :: 1 \text{ hour} = 3600'' : t'' = \frac{h \times 3600''}{m}$. Hence, to find the time of describing uB, we have, $h : h \times \text{sin. } ZB \times \text{cos. } CBZ \times \text{sec. } CBD \pm \frac{h^2 \times \text{sin. } ZB^2 \times \text{sin. } ZBD^2 \times \text{sec. } CBD^2}{AB} :: t : t \times \text{sin. } ZB \times \text{cos. } CBZ \times \text{sec. } CBD + \frac{t \times h \times \text{sin. } ZB^2 \times \text{sin. } ZBD^2 \times \text{sec. } CBD^2}{AB}$ the time of describing uB, or the effect of parallax in accelerating or retarding the time of contact; the upper sign is to be used when CBZ is acute, and the lower sign when it is obtuse. If CBZ be very nearly a right angle, but obtuse, it may happen that nE may be less than *no*, in which case, *nE* is to be taken from *no*, according to the rule. The principal part *nE* of the effect of parallax will increase or diminish the planet's distance from the sun's center, according as the angle *ZBC* is acute or obtuse; but the small part *no* of parallax will always increase the planet's distance from the center; take, therefore, the sum or difference of the effects, with the sign of the greater, as to increasing or decreasing the planet's distance from the center of the sun. The second part of the correction will not exceed 9″ or 10″ of time in the transits of Venus in 1761 and 1769, where the nearest approach of Venus to the sun's center was about 10′. In the transit of *Mercury*, the first part alone will be sufficient, except the nearest distance be much greater.

If we suppose the *mean* horizontal parallax of the sun to be 8″,83, then, by calculation from * the above expression, it appears that the total duration at Wardhus was lengthened by parallax 11′. 16″,88, and diminished at Otaheite by 12′. 10″,07; hence, the computed difference of the times is 23′. 26″,95; but the observed difference was 23′. 10″.

(332.) Hence, the correct parallax may be accurately found as follows. Because the observed difference of the total durations at Wardhus and Otaheite is 23′. 10″, and the computed difference, from the assumed mean horizontal parallax of the sun 8″,83, is 23′. 26″,95, the true parallax of the sun is less than that assumed. Let the true parallax be to that assumed as $1-e$ to 1, and (331) the first parts of the computed parallax will be lessened in the ratio of $1-e$: 1; and the second parts, in the ratio of $\overline{1-e}|^2$ to 1, or of $1-2e$ to 1 nearly. All the first parts, viz. 406″,05; 287″,05; 341″,48; 382″,47, in all = 1417,″05, combine the same way to make the total duration longer at Wardhus than at Otaheite. As to the second parts, the effects at Wardhus are $-7'',31$ and $-8'',91$, and at Otaheite are $+1'',63$ and $+4'',49$, in all $= -10'',10$. Therefore $1417'',05 \times \overline{1-e} - 10'',10 \times \overline{1-2e} = 1390''$ the excess of the observed total duration at Wardhus above that at Otaheite; or $1417'',05 - 10'',10 - 1390'' = \overline{1417'',05 - 20'',20} \times e$; and $e = \frac{16'',95}{1396'',85} = 0,0121$. Hence, the mean horizontal parallax of the sun $= 8'',83 \times \overline{1 - 0,0121} = 8'',72316$; we assume, therefore, the mean horizontal parallax of the sun $= 8''\frac{3}{4}$.

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* See my *Complete System of Astronomy*, Chap. 25.

Hence, the radius of the earth : the distance of the sun from the earth :: sin. $8''\frac{3}{4}$: rad. :: 1 : 23575.

(333.) The effect of the parallax being determined, the transit affords a very ready method of finding the difference of the longitudes of two places where the same observations were made. For, compute the effect of parallax in time, and reduce the observations at each place to the time, if seen from the center of the earth, and the difference of the times is the difference of the longitudes. For example, the times at Wardhus and Otaheite, at which the first internal contact would take place at the earth's center, are $9h.\ 40'.\ 44'',6$, and $21h.\ 38'.\ 25'',07$, the difference of which is $12h.\ 2'.\ 19'',53 = 180°.\ 34'.\ 53''$, the difference of the meridians. From the mean of 63 results from the transits of Mercury, Mr. *Short* found the difference of the meridians of Greenwich and Paris to be $9'.\ 15''$; and from the transit of Venus in 1761, to be $9'.\ 10''$, in time.

(334.) The transit of Venus affords a very accurate method of finding the place of the node. For by the observations made by Mr. *Rittenhouse*, at Norriton in the United States of America, the least distance CD was observed to be $10'.\ 10''$; hence, drawing CV perpendicular to $C☋$, cos. $DCV = 8°.\ 28'.\ 54''$: rad. :: $CD = 10'.\ 10''$: $CV = 10'.\ 17''$, the geocentric latitude of Venus at the time of conjunction; and * $0{,}72626 : 0{,}28895 :: 10'.\,17'' : 4'.\,5''$, the heliocentric latitude CV of Venus; hence, considering C ☊ V as a right lined triangle, tan. V ☊ $C = 3^\circ.\,23'.\,35''$: rad. :: the heliocentric latitude $CV = 4'.\,5''$: C ☊ $= 1^\circ.\,8'.\,52''$, which added to $2^s.\,13^\circ.\,26'.\,34''$, the place of the sun, gives $2^s.\,14^\circ.\,35'.\,26''$ for the place of the ascending node of the orbit of Venus.

(335.) The time of the ecliptic conjunction may be thus found. Find, at any time (t), the difference (d) of longitudes of Venus and the sun's center; find also the apparent geocentric horary motion (m) of Venus *from* the sun in longitude, and then say, $m : d ::$ 1 hour : the interval between the time (t) and the conjunction, which interval is to be added to or subtracted from t, according as the observation was made before or after the conjunction. In the transit in 1761, at $6h.\,31'.\,46''$, apparent time at Paris, M. *de la Lande* found $d = 2'.\,34'',4$, and $m = 3'.\,57'',4$; hence, $3'.\,57'',4 : 2'.\,34'',4 ::$ 1 hour : $39'.\,1''$, which subtracted from $6h.\,31'.\,46''$, because at that time the conjunction was past, gives $5h.\,52',\,45''$ for the time of conjunction from this observation. We may also thus find the latitude at conjunction. The horary motion of Venus in latitude was $35'',4$; hence, $60' : 39',1 :: 35'',4 : 23''$, the motion in latitude in $39',1$, which subtracted from $10'.\,1'',2$, the latitude observed at $6h.\,31'.\,46''$, gives $9'.\,38,''2$ for the latitude at the time of conjunction.

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* 0,72626 is the distance of Venus from the sun, her distance from the earth being 0,28895; and the angle subtended by CV is inversely as the distance from CV.

CHAP. XXIII.

ON THE NATURE AND MOTION OF COMETS.

(336.) COMETS are solid bodies, revolving in very excentric ellipses about the sun in one of the foci, and are therefore subject to the same laws as the planets, but differ in appearance from them; for as they approach the sun, a tail of light, in some of them, begins to appear, which increases till the comet comes to it's perihelion, and then it decreases again, and vanishes; others have a light encompassing the nucleus, or body of the comet, without any tail. The most ancient philosophers supposed comets to be like planets, performing their revolutions in stated times. *Aristotle*, in his first book of *Meteors*, speaking of comets, says, "But some of the Italians, called Pythagoreans, say, that a *Comet* is one of the *Planets*, but that they do not appear unless after a long time, and are seen but a small time, which happens also to *Mercury*." *Seneca* also, in *Nat. Quest.* Lib. vii. says, "*Apollonius* affirmed, that the *Comets* were, by the Chaldeans, reckoned among the *Planets*, and had their periods like them." *Seneca* himself also, having considered the phænomena of two remarkable comets, believed them to be stars of equal duration with the world, though he was ignorant of the laws that governed them; and foretold, that after-ages would unfold all these mysteries. He recommended it to Astronomers to keep a catalogue of the comets, in order to be able to determine whether they returned at certain periods. Notwithstanding this, most Astronomers, from his time till *Tycho Brahe*, considered them only as meteors, existing in our atmosphere. But that Astronomer, finding, from his own observations on a comet, that it had no diurnal parallax, placed them above the moon. Afterwards *Kepler* had an opportunity of observing two comets, one of which was very remarkable; and from his observations, which afforded sufficient indications of an annual parallax, he concluded, "that comets moved freely through the planetary orbs, with a motion not much different from a rectilinear one; but of what kind he could not precisely determine." *Hevelius* embraced the hypothesis of a rectilinear motion; but, finding his calculations did not perfectly agree with his observations, he concluded, "that the path of a comet was bent in a curve line, concave towards the sun." He supposed a comet to be generated in the atmosphere of a planet, and to be discharged from it, partly by the rotation of the planet, and then to revolve about the sun in a parabola by the force of projection and it's tendency to the sun, in the same manner as a projectile upon the earth's surface describes a parabola. At length came the famous comet in 1680, which descending nearly in a right line towards the sun, arose again from it in like manner, which proved it's motion in a curve about the sun. *G. S. Doerfell*, Minister at Plaven in Upper Saxony, made observations upon this comet, and found that it's motion might be very well represented by a parabola, having the sun in it's focus. He was ignorant, however, of all the laws by which the motion of a body in a parabola is regulated, and erred considerably in his parabola, making the perihelion distance about twelve times greater than it was. This was published five years before the *Principia*, in which work Sir *I. Newton* having proved that *Kepler*'s law, by which the motions of the planets are regulated, was a necessary consequence of his theory of gravity, it immediately followed, that comets were governed by the same law; and the observations upon them agreed so accurately with his theory, as to leave no doubt of it's truth. That comets describe ellipses, and not parabolas or hyperbolas, Dr. *Halley* (see his *Synopsis of the Astronomy of Comets*) advances the following reasons.

"Hitherto I have considered the orbits as exactly parabolic, upon which supposition it would follow, that comets, being impelled towards the sun by a centripetal force, would descend as from spaces infinitely distant, and, by their so falling, acquire such a velocity, as that they may again fly off into the remotest parts of the universe, moving upwards with a perpetual tendency, so as never to return again to the sun. But since they appear frequently enough, and since none of them can be found to move with an hyperbolic motion, or a motion swifter than what a comet might acquire by it's gravity to the sun, it is highly probable they rather move in very excentric elliptic orbits, and make their returns after long periods of time; for so their number will be determinate, and, perhaps, not so very great. Besides, the space between the sun and the fixed stars is so immense, that there is room enough for a comet to revolve, though the period of it's revolution be vastly long. Now, the *latus rectum* of an ellipsis is to the *latus rectum* of a parabola, which has the same distance in it's perihelion, as the distance in the aphelion, in the ellipsis, is to the whole axis of the ellipsis. And the velocities are in a subduplicate ratio of the same; wherefore, in very excentric orbits, the ratio comes very near to a ratio of equality; and the very small difference which happens, on account of the greater velocity in the parabola, is easily compensated in determining the situation of the orbit. The principal use, therefore, of the Table of the elements of their motions, and that which indeed induced me to construct it, is, that whenever a new comet shall appear, we may be able to know, by comparing together the elements, whether it be any of those which have appeared before, and consequently to determine it's period, and the axis of it's orbit, and to foretel it's return. And, indeed, there are many things which make me believe that the comet, which

Apian discovered in the year 1531, was the same with that which *Kepler* and *Longomontanus* more accurately described in the year 1607; and which I myself have seen return, and observed in the year 1682. All the elements agree, and nothing seems to contradict this my opinion, besides the inequality of the periodic revolutions; which inequality is not so great neither, as that it may not be owing to physical causes. For the motion of Saturn is so disturbed by the rest of the planets, especially Jupiter, that the periodic time of that planet is uncertain for some whole days together. How much more, therefore, will a comet be subject to such like errors, which rises almost four times higher than Saturn, and whose velocity, though increased but a very little, would be sufficient to change it's orbit, from an elliptic to a parabolical one. And I am the more confirmed in my opinion of it's being the same; for in the year 1456, in the summer-time, a comet was seen passing rotrograde between the earth and the sun, much after the same manner; which, though nobody made observations upon it, yet, from it's period, and the manner of it's transit, I cannot think different from those I have just now mentioned. And since looking over the histories of comets, I find, at an equal interval of time, a comet to have been seen about Easter in the year 1305, which is another double period of 151 years before the former. Hence, I think, I may venture to foretel that it will return again in the year 1758."

(337.) Dr. *Halley* computed the effect of *Jupiter* upon this comet in 1682, and found that it would increase it's periodic time above a year, in consequence of which he predicted it's return at the end of the year 1758, or the beginning of 1759. He did not make his computations with the utmost accuracy, but, as he himself informs us, *levi calamo*. M. *Clairaut* computed the effects both of Saturn and Jupiter, and found that the former would retard it's return in the last period 100 days, and the latter 511 days; and he determined the time when the comet would come to it's perihelion to be on April 15, 1759, observing that he might err a month, from neglecting small quantities in the computation. It passed the perihelion on March 13, within 33 days of the time computed. Now if we suppose the time stated by Dr. *Halley* to mean the time of it's passing the perihelion, then if we add to that 100 days, arising from the action of Saturn, which he did not consider, it will bring it very near to the time in which it did pass the perihelion, and prove his computation of the effect of Jupiter to have been very accurate. If he meant the time when it would first appear, his prediction was very accurate, for it was first seen on December 14, 1758, and his computation of the effects of Jupiter will then be more accurate than could have been expected, considering that he made his calculations only by an indirect method, and in a manner professedly not very accurate. Dr. *Halley*, therefore, had the glory, first to foretel the return of a comet, and the event answered remarkably to his prediction. He further observed, that the action of Jupiter, in the descent of the comet towards it's perihelion in 1682, would tend to increase the inclination of it's orbit; and accordingly the inclination in 1682 was found to be 22′ greater than in 1607. A learned Professor (Dr. *Long*'s Astronomy, p. 562) in Italy to an English gentleman, writes thus;—"Though M. *de la Lande*, and some other French gentlemen, have taken occasion to find fault with the inaccuracies of *Halley*'s calculation, because he himself had said, he only touched upon it slightly; nevertheless they can never rob him of the honour,—First, of finding out that it was one and the same comet which appeared in 1682, 1607, 1531, 1456, and 1305. —Secondly, of having observed, that the planet Jupiter would cause the inclination of the orbit of the comet to be greater, and the period longer.—Thirdly, of having foretold that the return thereof might be retarded till the end of 1758, or the beginning of 1759."

From the observations of M. *Messier* upon a comet in 1770, Mr. *Edric Prosperin*, Member of the Royal Academies of Stockholm and Upsal, showed, that a parabolic orbit would not answer to it's motions, and he recommended it to Astronomers to seek for the elliptic orbit. This laborious task M. *Lexell* undertook, and has shown that an ellipse, in which the periodic time is about five years and seven months, agrees very well with the observations. See the *Phil. Trans.* 1779. As the ellipses which the comets describe are very excentric, Astronomers, for the ease of calculation, suppose them to move in parabolic orbits, for that part which lies within the reach of observation, by which they can very accurately find the place of the perihelion; it's distance from the sun; the inclination of the plane of it's orbit to the ecliptic, and the place of the node. But it falls not within the plan of this work to enter into an investigation of these matters. For this, I refer the reader to my *Complete System of Astronomy*.

(338.) It is extremely difficult to determine, from computation, the elliptic orbit of a comet, to any degree of accuracy; for when the orbit is very excentric, a very small error in the observation will change the computed orbit into a parabola, or hyperbola. Now, from the thickness and inequality of the atmosphere with which the comet is surrounded, it is impossible to determine, with any great precision, when either the limb or center of the comet passes the wire at the time of observation. And this uncertainty in the observations will subject the computed orbit to a great error. Hence, it happened, that M. *Bouguer* determined the orbit of the comet to be an hyperbola. M. *Euler* first determined the same for the comet in 1774; but, having received more accurate observations, he found it to be an ellipse. The period of the comet in 1680 appears, from observation, to be 575 years, which Mr. *Euler*, by his computation, determined to be 166½ years. The only safe way to get the periods of comets, is to compare the elements of all those which have been computed, and where you find they agree very well, you may conclude that they are elements of the *same* comet, it being so extremely improbable that the orbits of two different comets should have the same inclination, the same perihelion distance, and the places of the perihelion and node the same. Thus, knowing the periodic time, we get the major axis of the ellipse; and the perihelion distance being known, the minor axis will be known. When the elements of the orbits agree, the comets may be the same, although the periodic times should vary a little; as that may arise from the attraction of the bodies in our system, and which may also alter all the other elements a little. We have already observed, that the comet which appeared in 1759, had it's periodic time increased considerably by the attraction of *Jupiter* and *Saturn*. This comet was seen in 1682, 1607, and 1531, all the elements agreeing, except a little variation of the periodic time. Dr. *Halley* suspected the comet in 1680, to have been the same which appeared in 1106, 531, and 44 years before CHRIST. He also conjectured, that the comet observed by *Apian*, in 1532, was the same as that observed by *Hevelius*, in 1661; if so, it ought to have returned in 1790, but it has never been observed. But M. *Mechain* having collected all the observations in 1532, and calculated the orbit again, found it to be sensibly different from that determined by Dr. *Halley*, which renders it very doubtful whether this was the comet which appeared in 1661; and this doubt is increased, by it's not appearing in 1790. The comet in 1770, whose periodic time M. *Lexell* computed to be five years and seven months, has not been observed since. There can be no doubt but that the path of this comet, for the time it was observed, belonged to an orbit whose periodic time was that found by M. *Lexell*, as the computations for such an orbit agreed so very well with the observations. But the revolution was probably longer before 1770; for as the comet passed very near to Jupiter in 1767, it's periodic time might be sensibly increased by the action of that planet; and as it has not been observed since, we may conjecture, with M. *Lexell*, that having passed in 1772 again into the sphere of sensible attraction of Jupiter, a new disturbing force might probably take place and destroy the effect of the other. According to the above elements, the comet would be in conjunction with Jupiter on August 23, 1779, and it's distance from Jupiter would be only $\frac{1}{419}$ of it's distance from the sun; consequently the sun's action would be only $\frac{1}{224}$ part of that of Jupiter. What a change must this make in the orbit! If the comet returned to it's perihelion in March, 1776, it would then not be visible. See M. *Lexell*'s account in the ***Phil. Trans.*** 1779. The elements of the orbits of the comets, in 1264 and 1556, were so nearly the same, that it is very probable it was the same comet; if so, it ought to appear again about the year 1848.

On the Nature and Tails of Comets.

(339.) Comets are not visible till they come into the planetary regions. They are surrounded with a very dense atmosphere, and from the side opposite to the sun they send forth a tail, which increases as the comet approaches it's perihelion, immediately after which it is longest and most luminous, and then it is generally a little bent and convex towards those parts to which the comet is moving; the tail then decreases, and at last it vanishes. Sometimes the tail is observed to put on this figure ∽ towards it's extremity, as that did in 1796. The smallest stars are seen through the tail, notwithstanding it's immense thickness, which proves that it's matter must be extremely rare. The opinion of the ancient philosophers, and of *Aristotle* himself, was, that the tail is a very thin fiery vapour arising from the comet. *Apian, Cardan, Tycho,* and others, believed that the sun's rays, being propagated through the transparent head of the comet, were refracted, as in a lens. But the figure of the tail does not answer to this; and, moreover, there should be some reflecting substance to render the rays visible, in like manner as there must be dust or smoke flying about in a dark room, in order that a ray of light entering it may be seen by a spectator standing sideways from it. *Kepler* supposed, that the rays of the sun carry away some of the gross parts of the comet which reflect the sun's rays, and give the appearance of a tail. *Hevelius* thought that the thinnest parts of the atmosphere of a comet are rarefied by the force of the heat, and driven from the fore part and each side of the comet towards the parts turned from the sun. Sir *I. Newton* thinks, that the tail of a comet is a very thin vapour, which the head, or nucleus of the comet, sends out by reason of it's heat. He supposes, that when a comet is descending to it's perihelion, the vapours behind the comet, in respect to the sun, being rarefied by the sun's heat, ascend, and take up with them the reflecting particles with which the tail is composed, as air rarefied by heat carries up the particles of smoke in a chimney. But as, beyond the atmosphere of the comet, the ætherial air (*aura ætherea*) is extremely rare, he attributes something to the sun's rays carrying with them the particles of the atmosphere of the comet. And when the tail is thus formed, it, like the nucleus, gravitates towards the sun, and by the projectile force received from the comet, it describes an ellipse about the sun, and accompanies the comet. It conduces also to the ascent of these vapours, that they revolve about the sun, and therefore endeavour to recede from it; whilst the atmosphere of the sun is either at rest, or moves with such a slow motion as it can acquire from the rotation of the sun about it's axis. These are the causes of the ascent of the tails in the neighbourhood of the sun, where the orbit has a greater curvature, and the comet moves in a denser atmosphere of the sun. The tail of the comet, therefore, being formed from the heat of the sun, will increase till it comes to it's perihelion, and decrease afterwards. The atmosphere of the comet is diminished as the tail increases, and is least immediately after the comet has passed it's perihelion, where it sometimes appears covered with a thick black smoke. As the vapour receives two motions when it leaves the comet, it goes on with the compound motion, and therefore the tail will not be turned directly from the sun, but decline from it towards those parts which are left by the comet; and meeting with a small resistance from the æther, will be a little curved. When the spectator, therefore, is in the plane of the comet's orbit, the curvature will not appear. The vapour, thus rarefied and dilated, may be at last scattered through the heavens, and be gathered up by the planets, to supply the place of those fluids which are spent in vegetation and converted into earth. This is the substance of Sir *I. Newton*'s account of the tails of comets. Against this opinion, Dr. *Hamilton*, in his *Philosophical Essays*, observes, that we have no proof of the existence of a solar atmosphere; and if we had, that when the comet is moving in it's perihelion in a direction at right angles to the direction of it's tail, the vapours which then arise, partaking of the great velocity of the comet, and being also specifically lighter than the medium in which they move, must suffer a much greater resistance than the dense body of the comet does, and therefore ought to be left behind, and would not appear opposite to the sun; and afterwards they ought to appear towards the sun. Also, if the splendour of the tails be owing to the reflection and refraction of the sun's rays, it ought to diminish the lustre of the stars seen through it, which would have their light reflected and refracted in like manner, and consequently their brightness would be diminished. Dr. *Halley*, in his description of the *Aurora Borealis* in

1716, says, "the streams of light so much resembled the long tails of comets, that at first sight they might well be taken for such." And afterwards, "this light seems to have a great affinity to that which the effluvia of electric bodies emit in the dark." *Phil. Trans.* N°. 347. *D. de Mairan* also calls the tail of a comet, the *aurora borealis* of the comet. This opinion Dr. *Hamilton* supports by the following arguments. A spectator, at a distance from the earth, would see the aurora borealis in the form of a tail opposite to the sun, as the tail of a comet lies. The aurora borealis has no effect upon the stars seen through it, nor has the tail of a comet. The atmosphere is known to abound with electric matter, and the appearance of the electric matter in vacuo is exactly like the appearance of the aurora borealis, which, from it's great altitude, may be considered to be in as perfect a vacuum as we can make. The electric matter in vacuo suffers the rays of light to pass through, without being affected by them. The tail of a comet does not expand itself sideways, nor does the electric matter. Hence, he supposes the tails of comets, the aurora borealis, and the electric fluid, to be matter of the same kind. We may add, as a further confirmation of this opinion, that the comet in 1607 appeared to shoot out at the end of it's tail. Le *P. Cysat* remarked the undulations of the tail of the comet in 1618. *Hevelius* observed the same in the tails of the comets in 1652 and 1661. M. *Pingre* took notice of the same appearance in the comet of 1769. These are circumstances exactly similar to the aurora borealis. Dr. *Hamilton* conjectures, that the use of the comets may be to bring the electric matter, which continually escapes from the planets, back into the planetary regions. The arguments are certainly strongly in favour of this hypothesis; and if this be true, we may further add, that the tails are hollow; for if the electric fluid only proceed in it's first direction, and do not diverge sideways, the parts directly behind the comet will not be filled with it; and this thinness of the tails will account for the appearance of the stars through them.

(340.) In respect to the nature of comets, Sir *I. Newton* observes, that they must be solid bodies like the planets; for if they were nothing but vapours, they must be dissipated when they come near the sun; for the comet in 1680, when it was in it's perihelion, was less distant from the sun than one-sixth of the sun's diameter, consequently the heat of the comet at that time was to the heat of the summer sun as 28000 to 1. But the heat of boiling water is about three times greater than the heat which dry earth acquires from the summer's sun; and the heat of red-hot iron about three or four times greater than the heat of boiling water. Therefore the heat of dry earth at the comet, when in it's perihelion, was about 2000 times greater than red-hot iron. By such heat, all vapours would be immediately dissipated.

(341.) This heat of the comet must be retained a very long time. For a red-hot globe of iron, of an inch diameter, exposed to the open air, scarce loses all it's heat in an hour; but a greater globe would retain it's heat longer, in proportion to it's diameter, because the surface, at which it grows cold, varies in that proportion less than the quantity of hot matter. Therefore a globe of red-hot iron, as large as our earth, would scarcely cool in 50000 years.

(342.) The comet in 1680, coming so near to the sun, must have been considerably retarded by the sun's atmosphere, and therefore, being attracted nearer at every revolution, it will at last fall into the sun, and be a fresh supply of fuel for what the sun loses by it's constant emission of light. In like manner, fixed stars which have been gradually wasted, may be supplied with fresh fuel, and acquire new splendor, and pass for new stars. Of this kind are those fixed stars which appear on a sudden, and shine with a wonderful brightness at first, and afterwards vanish by degrees. Such is the conjecture of Sir *I. Newton*.

(343.) From the beginning of our æra to this time, it is probable, according to the best accounts, that there have appeared about 500 comets. Before that time, about 100 others are recorded to have been seen, but it is probable that not above half of them were comets. And when we consider, that many others may not have been perceived, from being too near the sun—from appearing in moon-light—from being in the other hemisphere—from being too small to be perceived, or which may not have been recorded, we might imagine the whole number to be considerably greater; but it is likely, that of the comets which are recorded to have been seen, the same may have appeared several times, and therefore the number may be less than is here stated. The comet in 1786, which first appeared on August 1, was discovered by Miss *Caroline Herschel*, a sister of Dr. *Herschel*; since that time, she has discovered three others. As the plan of this work does not permit us to give the methods by which the orbits of comets may be computed, and all the opinions respecting them, if the reader wish to see any thing further on the subject, I refer him to my *Complete System of Astronomy*; or to Treatise, entitled *Cométographie, ou Traité Historique et Théorique des Cometès, par M.* PINGRE', *II. Tom. quarto. Paris,* 1784; or Sir H. ENGLEFIELD's *Determination of the Orbits of Comets,* a very valuable work, in which the ingenious Author has explained, with great clearness and accuracy, the manner of computing the orbits of comets, according to the methods of *Boscovich* and M. *de la Place.*

CHAP. XXIV.

ON THE FIXED STARS.

(344.) ALL the heavenly bodies beyond our system are called *Fixed Stars,* because (except some few) they do not appear to have any proper motion of their own. From their immense distance, they must be bodies of very great magnitude, otherwise they could not be visible; and when we consider the weakness of reflected light, there can be no doubt but that they shine with their own light. They are easily known from the planets, by their twinkling. The number of stars visible at once to the naked eye is about 1000; but Dr. *Herschel,* by his improvements of the reflecting telescope, has discovered that the whole number is great, beyond all conception. In that bright tract of the heavens, called the *Milky Way,* which, when examined by good telescopes, appears to be an immense collection of stars which gives that whitish appearance to the naked eye, he has, in a quarter of an hour, seen 116000 stars pass through the field of view of a telescope of only 15′ aperture. Every improvement of his telescopes has discovered stars not seen before, so that there appears to be no bounds to their number, or to the extent of the universe. These stars, which can be of no use to us, are probably suns to other systems of planets.

(345.) From an attentive examination of the stars with good telescopes, many, which appear only single to the naked eye, are found to consist of two, three, or more stars. Dr. *Maskelyne* had observed α *Herculis* to be a double star; Dr. *Hornsby* had found π *Bootis* to be double; M. *Cassini*, Mr. *Mayer*, Mr. *Pigott*, and many other Astronomers, had made discoveries of the like kind. But Dr. *Herschel*, by his improved telescopes, has found about 700, of which, not above 42 had been observed before. We shall here give an account of a few of the most remarkable.

α *Herculis*, FLAM. 64, a beautiful double star; the two stars very unequal, the largest is red, and the smallest blue, inclining to green.

δ *Lyræ*, FLAM. 12, double, very unequal, the largest red, and smallest dusky; not easily to be seen with a magnifying power of 227.

α *Geminorum*, FLAM. 66, double, a little unequal, both white; with a power of 146; their distance appears equal to the diameter of the smallest.

ε *Lyræ*, FLAM. 4 and 5, a double-double star; at first sight it appears double at a considerable distance, and, by a little attention, each will appear double; one set are equal, and both white; the other unequal, the largest white, and the smallest inclined to red. The interval of the stars, of the unequal set, is one diameter of the largest, with a power of 227.

γ *Andromedæ*, FLAM. 57, double, very unequal, the largest reddish white, the smallest a fine bright sky-blue, inclining to green. A very beautiful object.

α *Ursæ minoris*, FLAM. 1, double, very unequal, the largest white, the smallest red.

β *Lyræ*, FLAM. 10, quadruple, unequal, white, but three of them a little inclined to red.

α *Leonis*, FLAM. 32, double, very unequal, largest white, smallest dusky.

ε *Bootis*, FLAM. 36, double, very unequal, largest reddish, smallest blue, or rather a faint lilac; very beautiful.

b Draconis, FLAM. 39, a very small double star, very unequal, the largest white, smallest inclining to red.

λ *Orionis*, FLAM. 39, quadruple, or rather a double star, and has two more at a small distance, the double star considerably unequal, the largest white, smallest pale rose colour.

ξ *Libræ*, Flam. *ultima*, double-double, one set very unequal, the largest a very fine white.

μ *Cygni*, Flam. 78, double, considerably unequal, the largest white, the smallest blueish.

μ *Herculis*, Flam. 86, double, very unequal; the small star is not visible with a power of 278, but is seen very well with one of 460; the largest is inclined to a pale red, smallest duskish.

α *Capricorni*, Flam. 5, double, very unequal, the largest white, smallest dusky.

ν *Lyræ*, Flam. 8, treble, very unequal, the largest white, smallest both dusky.

α *Lyræ*, Flam. 3, double, very unequal, the largest a fine brilliant white, the smallest dusky; it appears with a power of 227. Dr. *Herschel* measured the diameter of this fine star, and found it to be 0″,3553.

(346.) These are a few of the principal double, &c. stars mentioned by Dr. *Herschel*, in his catalogues, which he has given us in the *Phil. Trans.* 1782 and 1785. The examination of double stars with a telescope, is a very excellent and ready method of proving it's powers. Dr. *Herschel* recommends the following method. The telescope and the observer having been some time in the open air, adjust the focus of the telescope to some single star of nearly the same magnitude, altitude, and colour of the star to be examined; attend to all the phænomena of the adjusting star as it passes through the field of view, whether it be perfectly round and well defined, or affected with little appendages playing about the edge, or any other circumstances of the like kind. Such deceptions may be detected by turning the object glass a little in it's cell, when these appendages will turn the same way. Thus you will detect the imperfections of the instrument, and therefore will not be deceived when you come to examine the double star.

(347.) Several stars, mentioned by ancient Astronomers, are not now to be found, and several are now observed, which do not appear in their catalogues. The most ancient observation of a new star, is that by *Hipparchus*, about 120 years before J. C. which occasioned his making a catalogue of the fixed stars, in order that future Astronomers might see what alterations had taken place since his time. We have no account where this new star appeared. A new star is also said to have appeared in the year 130; another in 389; another in the ninth century, in 15° of *Scorpio*; a fifth in 945; and a sixth in 1264; but the accounts we have of all these are very imperfect.

(348.) The first new star we have any accurate account of, is that which was discovered by *Cornelius Gemma*, on November 8, 1572, in the *Chair of Cassiopea.* It exceeded *Sirius* in brightness, and was seen at mid-day. It first appeared bigger than *Jupiter*, but it gradually decayed, and after sixteen months it entirely disappeared. It was observed by *Tycho Brahe*, who found that it had no sensible parallax; and he concluded that it was a fixed star. Some have supposed that this is the same which appeared in 945 and 1264, the situation of it's place favouring this opinion.

(349.) On August 13, 1596, *David Fabricius* observed a new star in the *Neck of the Whale*, in 25°. 45′ of Aries, with 15°. 54′ south latitude. It disappeared after October in the same year. *Phocyllides Holwarda* discovered it again in 1637, not knowing that it had ever been seen before; and after having disappeared for nine months, he saw it come into view again. *Bullialdus* determined the periodic time between it's greatest brightness to be 333 days. It's greatest brightness is that of a star of the second magnitude, and it's least, that of a star of the sixth. It's greatest degree of brightness, however, is not always the same, nor are the same phases always at the same interval.

(350.) In the year 1600, *William Jansenius* discovered a changeable star in the *Neck of the Swan.* It was seen by *Kepler*, who wrote a Treatise upon it, and determined it's place to be 16°. 18′ ♒, with 55°. 30′ or 32′ north latitude. *Ricciolus* saw it in 1616, 1621, 1624, and 1629. He is positive that it was invisible in the last years from 1640 to 1650. M. *Cassini* saw it again in 1655; it increased till 1660, and then grew less, and at the end of 1661, it disappeared. In November 1665, it appeared again, and disappeared in 1681. In 1715 it appeared of the sixth magnitude, as it does at present.

(351.) On June 20, 1670, another changeable star was discovered near the *Swan's Head*, by *P. Anthelme.* It disappeared in October, and was seen again on March 17, 1671. On September 11, it disappeared. It appeared again in March 1672, and disappeared in the same month, and has never since been seen. It's longitude was 1°. 52′. 26″ of ♒, and it's latitude 47°. 25′. 22″ N. The days are here put down for the new style.

(352.) In 1686, *Kirchius* observed χ in the *Swan* to be a changeable star; and, from twenty years observations, the period of the return of the same phases was found to be 405 days; the variations of it's magnitude, however, were subject to some irregularity.

(353.) In the year 1604, at the beginning of October, *Kepler* discovered a new star near the heel of the right foot of *Serpentarius*, so very brilliant, that it exceeded every fixed star, and even *Jupiter*, in magnitude. It was observed to be every moment changing into some of the colours of the rainbow, except when it was near the horizon, when it was generally white. It gradually diminished, and disappeared about October 1605, when it came too near the sun to be visible, and was never seen after. It's longitude was 17°. 40′ of ♐, with 1°. 56′ north latitude, and was found to have no parallax.

(354.) *Montanari* discovered two stars in the constellation of the *Ship*, marked β and γ by *Bayer*, to be wanting. He saw them in 1664, but lost them in 1668. The star θ in the tail of the *Serpent*, reckoned by *Tycho* of the third, was found, by him, of the fifth magnitude. The star ρ in *Serpentarius* did not appear, from the time it was observed by him, till 1695. The star ψ in the *Lion*, after disappearing, was seen by him in 1667. He observed also, that β in *Medusa*'s *Head* varied in it's magnitude.

(355.) M. *Cassini* discovered *one* new star of the fourth, and *two* of the fifth magnitude in *Cassiopea*; also *five* new stars in the same constellation, of which three have disappeared; *two* new ones in the beginning of the constellation *Eridanus*, of the fourth and fifth magnitude; and *four* new ones of the fifth or sixth magnitude, near the north pole. He further observed, that the star, placed by *Bayer* near ε of the *Little Bear*, is no longer visible; that the star A of *Andromeda*, which had disappeared, had come into view again in 1695; that in the same constellation, instead of one in the *Knee*, marked υ, there are two others come more northerly; and that ξ is diminished; that the star placed by *Tycho*, at the end of the *Chain of Andromeda*, as of the fourth magnitude, could then scarcely be seen; and that the star which, in *Tycho*'s catalogue, is the twentieth of *Pisces*, was no longer visible.

(356.) M. *Maraldi* observed, that the star κ in the left leg of *Sagittarius*, marked by *Bayer* of the third magnitude, appeared of the sixth, in 1671; in 1676 it was found, by Dr. *Halley*, to be of the third; in 1692 it could hardly be perceived, but in 1693 and 1694 it was of the fourth magnitude. In 1704 he discovered a star in *Hydra* to be periodical; it's position is in a right line with those in the tail marked π and γ. The time between it's greatest lustre, which is of the fourth magnitude, was about two years; in the intermediate time it disappeared. In 1666, *Hevelius* says, he could not find a star of the fourth magnitude in the eastern scale of *Libra*, observed by

Tycho and *Bayer*; but *Maraldi*, in 1709, says, that it had then been seen for 15 years, smaller than one of the fourth.

(357.) *J. Goodricke*, Esq. has determined the periodic variation of *Algol*, or β *Persei* (observed by *Montanari* to be variable) to be about 2*d.* 21*h.* It's greatest brightness is of the second magnitude, and least, of the fourth. It changes from the second to the fourth in about three hours and a half, and back again in the same time, and retains it's greatest brightness for the other part of the time.

(358.) Mr. *Goodricke* also discovered, that β *Lyræ* was subject to a periodic variation. The following is the result of his observations. It completes all it's phases in 12 days 19 hours, during which time, it undergoes the following changes:—1. It is of the third magnitude for about two days. 2. It diminishes in about 1¼ days. 3. It is between the fourth and fifth magnitude for less than a day. 4. It increases in about two days. 5. It is of the third magnitude for about three days. 6. It diminishes in about one day. 7. It is something larger than the fourth magnitude for a little less than a day. 8. It increases in about one day and three quarters to the first point, and so completes a whole period. See the *Phil. Trans.* 1785. He has also found, that δ *Cephei* is subject to a periodic variation of 5*d.* 8*h.* 37′½; during which time it undergoes the following changes: 1. It is at it's greatest brightness about one day thirteen hours. 2. It's diminution is performed in about one day eighteen hours. 3. It is at it's greatest obscuration about one day twelve hours. 4. It increases in about thirteen hours. It's greatest and least brightness is that between the third and fourth, and between the fourth and fifth magnitudes.

(359.) *E. Pigott*, Esq. has discovered η *Antinoi* to be a variable star, with a period of 7*d.* 4*h.* 38′. The changes happen as follows: 1. It is at it's greatest brightness 44 ± hours. 2. It decreases 62 ± hours.

3. It is at it's least brightness 30± hours. 4. It increases 36± hours. When most bright, it is of the third or fourth magnitude, and when least, of the fourth or fifth. See the *Phil. Trans.* 1785.

(360.) In the *Phil. Trans.* 1796, Dr. *Herschel* has proposed a method of observing the changes that may happen to the fixed stars; with a catalogue of their comparative brightness, in order to ascertain the permanency of their lustre.

(361.) Dr. *Herschel*, in a Paper of the *Phil. Trans.* 1783, upon the proper motion of the solar system, has given a large collection of stars which were formerly seen, but are now lost; also a catalogue of variable stars, and of new stars; and very justly observes, that it is not easy to prove that a star was never seen before; for though it should not be contained in any catalogue whatever, yet the argument for it's former non-appearance, which is taken from it's not having been observed before, is only so far to be regarded, as it can be made probable, or almost certain, that a star would have been observed, had it been visible.

(362.) There have been various conjectures to account for the appearances of the changeable stars. M. *Maupertuis* supposes, that they may have so quick a motion about their axes, that the centrifugal force may reduce them to flat oblate spheroids, not much unlike a mill-stone; and it's plane may be inclined to the plane of the orbits of it's planets, by whose attraction the position of the body may be altered, so that when it's plane passes through the earth, it may be almost or entirely invisible, and then become again visible as it's broadside is turned towards us. Others have conjectured, that considerable parts of their surfaces are covered with dark spots, so that when, by the rotation of the star, these spots are presented to us, the stars become almost or entirely invisible. Others have supposed, that these stars have very large opaque bodies revolving about and very near to them, so as to obscure them when they come in conjunction with us.

The irregularity of the phases of some of them shows the cause to be variable, and therefore may, perhaps, be best accounted for by supposing that a great part of the body of the star is covered with spots, which appear and disappear like those on the sun's surface. The total disappearance of a star may probably be the destruction of it's system; and the appearance of a new star, the creation of a new system of planets.

(363.) The fixed stars are not all evenly spread through the heavens, but the greater part of them are collected into clusters, of which it requires a large magnifying power, with a great quantity of light, to be able to distinguish the stars separately. With a small magnifying power and quantity of light, they only appear small whitish spots, something like a small light cloud, and thence they were called *Nebulæ*. There are some *nebulæ*, however, which do not receive their light from stars. For in the year 1656, *Huygens* discovered a nebula in the middle of *Orion's Sword*; it contains only seven stars, and the other part is a bright spot upon a dark ground, and appears like an opening into brighter regions beyond. In 1612, *Simon Marius* discovered a nebula in the *Girdle of Andromeda*. Dr. *Halley*, when he was observing the southern stars, discovered one in the *Centaur*, but this is never visible in England. In 1714, he found another in *Hercules*, nearly in a line with ζ and η of *Bayer*. This shows itself to the naked eye, when the sky is clear and the moon absent. M. *Cassini* discovered one between the *Great Dog* and the *Ship*, which he describes as very full of stars, and very beautiful, when viewed with a good telescope. There are two whitish spots near the south pole, called, by the sailors, the *Magellanic Clouds*, which, to the naked eye, resemble the milky way, but, through telescopes, they appear to be composed of stars. M. *de la Caille*, in his catalogue of fixed stars observed at the Cape of Good Hope, has remarked 42 nebulæ which he observed, and which he divided into three classes; 14, in which he could not discover the stars; 14, in which he could see a distinct mass of stars; and 14, in which the stars appeared of the sixth magnitude, or below, accompanied with white spots, and nebulæ of the first and third kind. In the *Connoissance des Temps*, for 1783 and 1784, there is a catalogue of 103 nebulæ, observed by *Messier* and *Mechain*, some of which they could resolve, and others they could not. But Dr. *Herschel* has given us a catalogue of 2000 nebulæ and clusters of stars, which he himself has discovered. Some of them form a round, compact system; others are more irregular, of various forms; and some are long and narrow. The globular systems of stars appear thicker in the middle than they would do if the stars were all at equal distances from each other; they are, therefore, condensed towards the center. That the stars should be thus accidentally disposed, is too improbable a supposition to be admitted; he supposes, therefore, that they are thus brought together by their mutual attractions, and that the gradual condensation towards the center is a proof of a central power of that kind. He further observes, that there are some additional circumstances in the appearance of extended clusters and nebulæ, that very much favour the idea of a power lodged in the brightest part. For although the form of them be not globular, it is plainly to be seen that there is a tendency towards sphericity, by the swell of the dimensions as they draw near the most luminous place, denoting, as it were, a course, or tide of stars, setting towards a center. As the stars in the same nebula must be very nearly all at the same relative distance from us, and they appear nearly of the same size, their real magnitudes must be nearly equal. Granting, therefore, that these nebulæ and clusters of stars are formed by their mutual attraction, Dr. *Herschel* concludes that we may judge of their relative age by the disposition of their component parts, those being the oldest which are most compressed. He supposes the milky way to be a nebula, of which our sun is one of it's component parts. See the *Phil. Trans.* 1786 and 1789.

(364.) Dr. *Herschel* has discovered other phænomena in the heavens, which he calls *Nebulous Stars*; that is, stars surrounded with a faint luminous atmosphere, of a considerable extent. Cloudy or nebulous stars, he observes, have been mentioned by several Astronomers; but this name ought not to be applied to the objects which they have pointed out as such; for, on examination, they prove to be either clusters of stars, or such appearances as may reasonably be supposed to be occasioned by a multitude of stars at a vast distance. He has given an account of seventeen of these stars, one of which he has thus described. "November 13, 1790. A most singular phænomenon; a star of the eighth magnitude, with a faint luminous atmosphere, of a circular form, and of about 3′ diameter. The star is perfectly in the center, and the atmosphere is so diluted, faint, and equal throughout, that there can be no surmise of it's consisting of stars; nor can there be a doubt of the evident connexion between the atmosphere and the star. Another star not much less in brightness, and in the same field of view with the above, was perfectly free from any such appearance." Hence, he draws the following consequences. Granting the connexion between the star and the surrounding nebulosity, if it consist of stars very remote which give the nebulous appearance, the central star, which is visible, must be immensely greater than the rest; or if the central star be not larger than common, how extremely small and compressed must be those other luminous points which occasion the nebulosity! As, by the former supposition, the luminous central point must far exceed the standard of what we call a star, so, in the latter, the shining matter about the center will be much too small to come under the same denomination; we therefore either have a central body which is not a star, or a star which is involved in a shining fluid, of a nature totally unknown to us. This last opinion Dr. *Herschel* adopts. The existence of this shining matter, he says, does not seem to be so essentially connected with the central points, that it might not exist without them. The great resemblance there is between the chevelure of these stars, and the diffused nebulosity there is about the constellation *Orion*, which takes up a space of more than 60 square degrees, renders it highly probable that they are of the same nature. If this be admitted, the separate existence of the luminous matter is fully proved. Light reflected from the star could not be seen at this distance. And, besides, the outward parts are nearly as bright as those near the star. In further confirmation of this, he observes, that a cluster of stars will not so completely account for the milkiness, or soft tint of the light of these nebulæ, as a self-luminous fluid. This luminous matter seems more fit to produce a star by it's condensation, than to depend on the star for it's existence. There is a telescopic milky way extending in right ascension from 5*h*. 15′. 8″ to 5*h*. 39′. 1″, and in polar distance from 87°. 46′ to 98°. 10′. This, Dr. *Herschel* thinks, is better accounted for, by a luminous matter, than from a collection of stars. He observes, that perhaps some may account for these nebulous stars, by supposing that the nebulosity may be formed by a collection of stars at an immense distance, and that the central star may be a near star, accidentally so placed; the appearance, however, of the luminous part does not, in his opinion, at all favour the supposition that it is produced by a great number of stars; on the other hand, it must be granted that it is extremely difficult to admit the other supposition, when we know that nothing but a solid body is self-luminous, or, at least, that a fixed luminary must immediately depend upon such, as the flame of a candle upon the candle itself. See Dr. *Herschel*'s Account, in the *Phil. Trans.* 1791.

On the Constellations.

(365.) As soon as Astronomy began to be studied, it must have been found necessary to divide the heavens into separate parts, and to give some representations to them, in order that Astronomers might describe and speak of the stars, so as to be understood. Accordingly we find that these circumstances took place very early. The ancients divided the heavens into *Constellations*, or collections of stars, and represented them by animals, and other figures; according to the ideas which the dispositions of the stars suggested. We find some of them mentioned by JOB; and although it has been disputed, whether our translation has sometimes given the true meaning to the Hebrew words, yet it is agreed, that they signify constellations. Some of them are mentioned by *Homer* and *Hesiod*, but *Aratus* professedly treats of all the ancient ones, except three which were invented after his time. The number of the ancient constellations was 48, but the present number upon a globe is about 70; by rectifying which, and setting it to correspond with the stars in the heavens, you may, by comparing them, very easily get a knowledge of the different constellations and stars. Those stars which do not come into any of the constellations, are called *Unformed Stars*. The stars visible to the naked eye are divided into six classes, according to their magnitudes; the largest are called of the first magnitude, the next of the second, and so on. Those which cannot be seen without telescopes, are called *Telescopic Stars*. The stars are now generally marked upon maps and globes with *Bayer*'s letters; the 1st letter in the Greek alphabet being put to the greatest star of each constellation; the 2d letter to the next greatest, and so on; and when any more letters are wanted, the Italic letters are generally used; this serves as a name to the star, by which it may be pointed out. Twelve of these constellations lie upon the ecliptic, including a space about 15° broad, called the *Zodiac*, within which all the planets move. The constellation *Aries*, or the *Ram*, about 2000 years since, lay in the *first* sign of the ecliptic; but, on account of the precession of the equinox, it now lies in the *second*. The following are the names of the constellations, and the number of the stars observed in them by different Astronomers. *Antinous* was made out of the unformed stars near *Aquila*; and *Coma Berenices* out of the unformed stars near the *Lion's Tail*. They are both mentioned by *Ptolemy*, but as unformed stars. The constellations as far as the *Triangle*, with *Coma Berenices*, are *northern*; those after *Pisces*, are *southern*.

THE ANCIENT CONSTELLATIONS.		*Ptolemy.*	*Tycho.*	*Hevelius.*	*Flamstead.*
Ursa Minor	The Little Bear	8	7	12	24
Ursa Major	The Great Bear	35	29	73	87
Draco	The Dragon	31	32	40	80
Cæpheus	Cæpheus	13	4	51	35
Bootes	Bootes	23	18	52	54
Corona Borealis	The Northern Crown	8	8	8	21
Hercules	Hercules kneeling	29	28	45	113
Lyra	The Harp	10	11	17	21
Cygnus	The Swan	19	18	47	81
Cassiopea	The Lady in her Chair	13	26	37	55
Perseus	Perseus	29	29	46	59
Auriga	The Waggoner	14	9	40	66
Serpentarius	Serpentarius	29	15	40	74
Serpens	The Serpent	18	13	22	64
Sagitta	The Arrow	5	5	5	18
Aquila	The Eagle	15	12	23	71
Antinous	Antinous		3	19	
Delphinus	The Dolphin	10	10	14	18

THE ANCIENT CONSTELLATIONS CONTINUED.

		Ptolemy.	*Tycho.*	*Hevelius.*	*Flamstead.*
Equulus	The Horse's Head	4	4	6	10
Pegasus	The Flying Horse	20	19	38	89
Andromeda	Andromeda	23	23	47	66
Triangulum	The Triangle	4	4	12	16
Aries	The Ram	18	21	27	66
Taurus	The Bull	44	43	51	141
Gemini	The Twins	25	25	38	85
Cancer	The Crab	23	15	29	83
Leo	The Lion	35	30	49	95
Coma Berenices	Berenice's Hair		14	21	43
Virgo	The Virgin	32	33	50	110
Libra	The Scales	17	10	20	51
Scorpius	The Scorpion	24	10	20	44
Sagittarius	The Archer	31	14	22	69
Capricornus	The Goat	28	28	29	51
Aquarius	The Water-bearer	45	41	47	108
Pisces	The Fishes	38	36	39	113
Cetus	The Whale	22	21	45	97
Orion	Orion	38	42	62	78
Eridanus	Eridanus	34	10	27	84
Lepus	The Hare	12	13	16	19
Canis Major	The Great Dog	29	13	21	31
Canis Minor	The Little Dog	2	2	13	14
Argo	The Ship	45	3	4	64
Hydra	The Hydra	27	19	31	60
Crater	The Cup	7	3	10	31
Corvus	The Crow	7	4	0	9
Centaurus	The Centaur	37	0	0	35
Lupus	The Wolf	19	0	0	24
Ara	The Altar	7	0	0	9
Corona Australis	The Southern Crown	13	0	0	12
Piscis Australis	The Southern Fish	18	0	0	24

THE NEW SOUTHERN CONSTELLATIONS.

Columba Noachi	Noah's Dove	10
Robur Carolinum	The Royal Oak	12
Grus	The Crane	13
Phœnix	The Phœnix	13
Indus	The Indian	12
Pavo	The Peacock	14
Apus, *Avis Indica*	The Bird of Paradise	11
Apis, *Musca*	The Bee, or Fly	4
Chamæleon	The Chameleon	10
Triangulum Australe	The South Triangle	5
Piscis volans, *Passer*	The Flying Fish	8
Dorado, *Xiphias*	The Sword Fish	6
Toucan	The American Goose	9
Hydrus	The Water Snake	10

HEVELIUS's CONSTELLATIONS,

MADE OUT OF THE UNFORMED STARS.

Lynx	The Lynx	19	44
Leo Minor	The Little Lion		53
Asteron and Chara	The Greyhounds	23	25
Cerberus	Cerberus	4	
Vulpecula and Anser	The Fox and Goose	27	35
Scutum Sobieski	Sobieski's Shield	7	
Lacerta	The Lizard		16
Camelopardalis	The Camelopard	32	58
Monoceros	The Unicorn	19	31
Sextans	The Sextant	11	41

Besides the letters which are prefixed to the stars, many of them have names, as *Regulus*, *Syrius*, *Arcturus*, &c.

(366.) *Kepler*, who was afterwards, in this conjecture, followed by Dr. *Halley*, has made a very ingenious observation upon the magnitudes and distances of the fixed stars. He observes, that there can be only 13 points upon the surface of a sphere as far distant from each other as from the center; and supposing the nearest fixed stars to be as far from each other as from the sun, he concludes that there can be only thirteen stars of the first magnitude. Hence, at twice that distance from the sun, there may be placed four times as many, or 52; at three times that distance, nine times as many, or 117; and so on. These numbers will give, pretty nearly, the number of stars of the first, second, third, &c. magnitudes. Dr. *Halley* further remarks, that if the number of stars be finite, and occupy only a part of space, the outward stars would be continually attracted towards those which are within, and, in process of time, they would coalesce and unite into one. But if the number be infinite, and they occupy an infinite space, all the parts would be nearly in equilibrio, and consequently, each fixed star being drawn in opposite directions, would keep it's place, or move on till it had found an equilibrium. *Phil. Trans.* N°. 364.

On the Catalogues of the Fixed Stars.

(367.) At the time of *Hipparchus* of Rhodes, about 120 years before J. C. a new star appeared, upon which he set about numbering the fixed stars, and reducing them to a *Catalogue,* that posterity might know whether any changes had taken place in the heavens. *Ptolemy,* however, mentions that *Tymocharis* and *Arystillus* left several observations made 180 years before. The catalogue of *Hipparchus* contained 1022 stars, with their latitudes and longitudes, which *Ptolemy* published, with the addition of four more. These Astronomers made their observations with an armillary sphere, placing the armilla, or hoop representing the ecliptic, to coincide with the ecliptic in the heavens by means of the sun in the day-time, and then they determined the place of the moon in respect of the sun by a moveable circle of latitude. The next night, by the help of the moon (whose place before found they corrected by allowing for it's motion in the interval of time) they placed the hoop in such a situation as was agreeable to the present moment of time, and then compared, in like manner, the places of the stars with the moon. Thus they found the latitudes and longitudes of the stars; it could not, however, be done with such an instrument to any very great degree of accuracy. *Ptolemy* adapted his catalogue to the year 137 after J. C.; but supposing, with *Hipparchus*, who made the discovery, the precession of the equinoxes to 1° in 100 years, instead of about 72 years, he only added 2°. 40′ to the numbers in *Hipparchus* for 265 years (the difference of the epochs) instead of 3°. 42′. 22″, according to Dr. *Maskelyne*'s Tables. To compare his Tables, therefore, with the present, we must first increase his numbers by 1°. 2′. 22″, and then allow for the precession from that time to this. The next Astronomer who observed the fixed stars a-new, was *Ulugh Beigh*, the grandson of *Tamerlane* the Great; he made a catalogue of 1022 stars, reduced to the year 1437. *William*, the most illustrious Landgrave of Hesse, made a catalogue of 400 stars which he observed; he computed their latitudes and longitudes from their observed right ascensions and declinations. In the year 1610, *Tycho Brahe*'s catalogue of 777 stars was published from his own observations, made with great care and diligence. It was afterwards, in 1627, copied into the *Rudolphine Tables*, and increased by 223 stars, from other observations of *Tycho*. Instead of a *zodiacal* armilla, *Tycho* substituted the *equatorial* armilla, by which he observed the difference of right ascensions, and the declinations, out of the meridian, the meridian altitude being always made use of to confirm the others. From thence he computed the latitudes and longitudes. *Tycho* compared *Venus* with the sun, and then the other stars with Venus, allowing for it's parallax and refraction; and having thus ascertained the places of a few stars, he settled the rest from them; and although his instrument was very large, and constructed with great accuracy, yet, not having pendulum clocks to measure his time, his observations cannot be very accurate. The next catalogue was that of *R. P. Ricciolus*, which was taken from *Tycho*'s, except 101 stars which he himself had observed. *Hevelius* of Dantzick, in 1690, published a catalogue of 1930 stars, of which 950 were known to the ancients; 603 he calls his own, because they had not been accurately observed by any one before himself; and 377 of Dr. *Halley*, which were invisible to his hemisphere. Their places were fixed for the year 1660. The *British Catalogue*, which was published by Mr. *Flamstead*, contains 3000 stars, rectified for the year 1689. They are distinguished into seven degrees of magnitude (of which the seventh degree is telescopic) in their proper constellations. This catalogue is more correct than any of the others, the observations having been made with better instruments. He also published an *Atlas Cœlestis*, or maps of the stars, in which each star is laid down in it's true place, and delineated of it's own magnitude. Each star is marked with a letter, beginning with the first letter α of the Greek alphabet for the largest star of each constellation, and so on, according to their magnitudes, following, in this respect, the charts of the same kind which were published by *J. Bayer*, a German, 1603. In the year 1757, M. *de la Caille* published his *Fundamenta Astronomiæ*, in which there is a catalogue of 397 stars; and in 1763, he published a catalogue of 1942 southern stars, from the tropic of Capricorn to the south pole, with their right ascensions and declinations for 1750. He also published a catalogue of zodiacal stars in the *Ephemerides* from 1765 to 1774. Mr. *Mayer* also published a catalogue of 600 zodiacal stars. In the *Nautical Almanac* for 1773, there is published a catalogue of

380 stars observed by Dr. *Bradley*, with their longitudes and latitudes. In the year 1782, *J. E. Bode*, Astronomer at Berlin, published a set of *Celestial Charts*, containing a greater number of stars than in those of Mr. *Flamstead*, with many of the double stars and nebulæ. He also published, in the same work, a catalogue of stars, that of *Flamstead* being the foundation, omitting some stars, whose positions were left incomplete, and altering the numbers; to which he has added stars from *Hevelius*, M. *de la Caille*, *Mayer*, and others. In the year 1776, there was published at Berlin, a work entitled *Recueil de Tables Astronomiques*, in which is contained a very large catalogue of stars from *Hevelius*, *Flamstead*, M. *de la Caille*, and Dr. *Bradley*, with their latitudes and longitudes for the beginning of 1800; with a catalogue of the southern stars of M. *de la Caille*; of double stars; of changeable stars, and of nebulous stars. This is a very useful work for the practical Astronomer. But the most complete catalogue is that published by the Rev. Mr. *Wollaston*, F. R. S. in 1789, entitled, *A Specimen of a General Astronomical Catalogue, arranged in Zones of North Polar Distance, and adapted to January* 1, 1790; *containing a Comparative View of the Mean Positions of Stars, as they come out upon Calculation from the Tables of several principal Observers.*

On the Proper Motions *of the Fixed Stars.*

(368.) Dr. *Maskelyne*, in the explanation and use of his Tables, which he published with the first volume of his *Observations*, observes, that many, if not all the fixed stars, have small motions among themselves, which are called their *Proper Motions*; the cause and laws of which are hid, for the present, in almost equal obscurity. From comparing his own observations at that time, with those of Dr. *Bradley*, Mr. *Flamstead*, and Mr. *Roemer*, he then found the annual proper motion of the following stars, in right ascension, to be, of *Sirius* – 0″,63, of *Castor* – 0″,28, of *Procyon* – 0″,8, of *Pollux* – 0″,93, of *Regulus* – 0″,41, of *Arcturus* – 1″,4, and of α *Aquilæ* + 0″,57; and of *Sirius* in north polar distance 1″,20, and of *Arcturus* 2″,01, both southwards. But since that time he had continued his observations, and from a catalogue of right ascensions of 36 principal stars (which he communicated to Mr. *Wollaston*, and which is found in his work), it appears that 35 of them have a *proper motion* in right ascension.

(369.) In the year 1759, M. *Mayer* observed 80 stars, and compared them with the observations of *Roemer* in 1706. M. *Mayer* is of opinion, that (from the goodness of the instruments with which the observations were made) where the disagreement is at least 10″ or 15″, it is a very probable indication of a proper motion of such a star. He further adds, that when the disagreement is so great as he has found it in some of the stars, amongst which is *Fomahand*, where the difference was 21″ in 50 years, he has no doubt of a proper motion. Dr. *Herschel*, following *Mayer*'s judgement of his own and *Roemer*'s observations, has compared the observations, and leaving out of his account all those stars which did not show a disagreement amounting to 10″, he found that 56 of them had a proper motion. From thence he attempts to deduce the motion of the solar system in the following manner.

(370.) If the sun be in motion as well as the stars, the effects will be altered according to their motion, compared with the motion of our sun. Some of them, therefore, from their own proper motions, might destroy, or more than counteract, the effects arising from the motion of the sun. In whatever direction our system should move, it would be very easy to find what effect in latitude and longitude would have taken place upon any star, by means of a celestial globe, by conceiving the sun to move from the center upon any radius directed to the point to which the sun is moving. Dr. *Herschel* describes the effect thus. Let an arc of 90° be applied to the surface of a globe, and always passing through that point to which the motion of the system is directed. Then whilst one end moves along the equator, the other will describe a curve passing through it's pole, and returning into itself; and the stars in the northern hemisphere, within this curve, will appear to move to the north; and the rest will go to the south. A similar curve may be described in the southern hemisphere, and like appearances will take place.

(371.) Now Dr. *Herschel* first takes the seven stars before mentioned, whose proper motions had been determined by Dr. *Maskelyne*, and he finds, that if a point be assumed about the 77° of right ascension, and the sun to move from it, it will account for all the motions in right ascension. And if, instead of supposing the sun to move in the plane of the equator, it should ascend to a point near to λ *Herculis*, it will account for the observed change of declination of *Sirius* and *Arcturus*. In respect to the *quantity* of motion of each, that must depend upon their unknown relative distances; he only speaks here of the *directions* of the motions.

(372.) He next takes twelve stars from the catalogue of 56, whose proper motions have been determined from a comparison of the observations of *Roemer* and *Mayer*, and adds to them *Regulus* and *Castor*; these have all a proper motion in right ascension and declination, except Regulus, which has none in declination. Of these 27 motions, the above-supposed motion of the solar system will satisfy 22. There are also some remarkable circumstances in the *quantities* of these motions. *Arcturus* and *Sirius* being the largest, and therefore, probably, the nearest, ought to have the greatest apparent motion; and so we find they have. Also, Arcturus is better situated to have a motion in right ascension, and it has the greatest motion. Several other facts of the same kind are found also to take place. But there is a very remarkable circumstance in respect to Castor. Castor is a double star; now, how extraordinary must appear the concurrence, that two such stars should both have a proper motion so exactly alike, that they never have been found to vary a second! This seems to point out the common cause, the motion of the solar system.

(373.) Dr. *Herschel* next takes 32 more of the same catalogue of 56 stars, and shows that their motions agree very well with his supposed motion of the solar system. But the motions of the other 12 stars cannot be accounted for upon this hypothesis. In these, therefore, he supposes the effect of the solar motion has been destroyed and counteracted by their own proper motions. The same may be said of 19 stars out of the 32, which only agree with the solar motion one way, and are, as to sense, at rest in the other. According to the rules of philosophising, therefore, which direct us to refer all phænomena to as few and simple principles as are sufficient to explain them, Dr. *Herschel* thinks we ought to admit the motion of the solar system. Perhaps, however, this argument cannot be properly applied here, because there is no new cause or principle introduced, by supposing each star to have a proper motion. Admitting the doctrine of universal gravitation, the fixed stars ought to move as well as the sun. But the sun's motion, as here estimated, cannot be owing to the action of a body upon it which might give it a rotatory motion at the same time, as M. *de la Lande* conjectures; because a body acting on the sun, to give it it's rotation about it's axis, would not, at the same time, give it that progressive motion. See Dr. *Herschel's* Account in the *Phil. Trans.* 1783.

(374.) But it will be proper to consider, how far this motion of the solar system agrees with the proper motion of the 35 stars determined by Dr. *Maskelyne*.

Now, upon supposition that the sun moves, as conjectured by Dr. *Herschel,* that motion will account for the motion of 20 of them, so far as regards their directions; but the motion of the other 15 is contrary to that which ought to arise from this supposition. As some of the stars must have a proper motion of their own, even upon the hypothesis of a solar motion, and which probably arises from their mutual attraction, it is very probable that they have all a proper motion from the same cause, but most of them so very small, as not yet to have been discovered. And it might also happen, that such a motion might be the same as that which would arise from the motion of the solar system. Yet it must be confessed, that the circumstance of *Castor,* and the motions, both in right ascension and declination, of many of the stars being such as arise from this hypothesis, with the apparent motion of those stars being greatest which are probably nearest, form a strong argument in it's favour.

On the Zodiacal Light.

(375.) The *Zodiacal Light* is a pyramid of light which sometimes appears in the morning before sun-rise, and in the evening after sun-set. It has the sun for it's basis, and in appearance resembles the *Aurora Borealis.* It's sides are not straight, but a little curved, it's figure resembling a lens edgeways. It is generally seen here about October and March, that being the time of our shortest twilight; for it cannot be seen in the twilight; and when the twilight lasts a considerable time, it is withdrawn before the twilight ends. It was observed by M. *Cassini,* in 1683, a little before the vernal equinox, in the evening, extending along the ecliptic from the sun. He thinks, however, that it has appeared formerly, and afterwards disappeared, from an observation of Mr. *J. Childrey,* in a book published in 1661, entitled, *Britannia Baconia.* He says, that " in the month of February, for several years, about six o'clock in the evening, after twilight, he saw a path of light tending from the twilight towards the *Pleiades,* as it were touching them. This is to be seen whenever the weather is clear, but best when the moon does not shine. I believe this phænomenon has been formerly, and will hereafter appear always at the above-mentioned time of the year. But the cause and nature of it I cannot guess at, and therefore leave it to the enquiry of posterity." From this description, there can be no doubt but that this was the zodiacal light. He suspects also, that this is what the aucients called *Trabes,* which word they used for a meteor, or impression in the air like a beam. *Pliny,* lib. II. p. 26, says, *Emicant Trabes, quos docos vocant. Des Cartes* also speaks of a phænomenon of the same kind. M. *Fatio de Duillier* observed it immediately after the discovery by M. *Cassini,* and suspected that it had always appeared. It was soon after observed by M. *Kirch* and *Eimmart* in Germany. In the year 1707, on April 3, it was observed by Mr. *Derham,* in Essex. It appeared in the western part of the heavens, about a quarter of an hour after sun-set, in the form of a pyramid, perpendicular to the horizon. The base of this pyramid he judged to be the sun. It's vertex reached 15° or 20° above the horizon. It was throughout of a dusky red colour, and at first appeared pretty vivid and strong, but faintest at the top. It grew fainter by degrees, and vanished about an hour after sun-set. This solar atmosphere has also been seen about the sun in a total solar eclipse, a luminous ring appearing about the moon at the time when the eclipse was total.

(376.) M. *Fatio* conjectured, that this appearance arises from a collection of corpuscles encompassing the sun in the form of a lens, reflecting the light of the sun. M. *Cassini* supposed that it might arise from an infinite number of planets revolving about the sun; so that this light might owe it's existence to these bodies, as the milky way does to an innumerable number of fixed stars. It is now, however, generally supposed, that it is matter detached from the sun by it's rotation about it's axis. The velocity of the equatorial parts of the sun being the greatest, would throw the matter to the greatest distance, and, on account of the diminution of velocity towards it's poles, the height to which the matter would there rise would be diminished; and as it would probably spread a little sideways, it would form an atmosphere about the sun something in the form of a lens, whose section perpendicular to it's axis would coincide with the sun's equator. And this agrees very well with observation. There is, however, a difficulty in thus accounting for this phænomenon. It is very well known that the centrifugal force of a point of the sun's equator is a great many times less than it's gravity. It does not appear, therefore, how the sun, from it's rotation, can detach any of it's gross particles. If they be particles detached from the sun, they must be sent off by some other unknown force; and in that case they might be sent off equally in all directions, which would not agree with the observed figure. The cause is probably owing to the sun's rotation, although not immediately to the centrifugal force arising therefrom.

CHAP. XXV.

ON THE LONGITUDE OF PLACES UPON THE SURFACE OF THE EARTH.

(377.) THE situation of any place upon the earth's surface is determined from it's latitude and longitude. The latitude may be found from the meridian altitude of the sun, or a known fixed star; from two altitudes of the sun, and the time between; and by a variety of other methods. These operations are so easy in practice, and opportunities are so continually offering themselves, that the latitude of a place may generally be determined, even under the most unfavourable circumstances, to a degree of accuracy sufficient for all nautical purposes. But the longitude cannot be so readily found. *Philip* III. King of Spain, was the first person who offered a reward for it's discovery; and the States of Holland, soon after, followed his example, they being at that time rivals to Spain, as a maritime power. During the minority of *Louis* XV. of France, the regent power promised a great reward to any person who should discover the longitude at sea. In the time of *Charles* II. about 1675, the Sieur *de St. Pierre*, a Frenchman, proposed a method of finding the longitude by the moon. Upon this, a commission was granted to Lord Viscount *Brounker*, President of the Royal Society, Mr. *Flamstead*, and several others, to receive his proposals, and give opinion respecting it. Mr. *Flamstead* gave his opinion, that if we had Tables of the places of the fixed stars, and of the moon's motions, we might find the longitude, but not by the method proposed by the Sieur *de St. Pierre.* Upon this, Mr. *Flamstead* was appointed Astronomer Royal, and an Observatory was built at Greenwich for him; and the instructions to him and his successors were, "That they should apply themselves with the utmost care and diligence to rectify the Tables of the motions of the heavens, and the places of the fixed stars, in order to find out the so-much desired longitude at sea, for the perfecting of the art of navigation."

(378.) In the year 1714, the British Parliament offered a reward for the discovery of the longitude; the sum of 10000*l.* if the method determined the longitude to 1° of a great circle, or 60 geographical miles; of 15000*l.* if it determined it to 40 miles; and of 20000*l.* if it determined it to 30 miles; with this proviso, that if any such method extend no further than 80 miles adjoining to the coast, the proposer shall have no more than half such rewards*. The Act also appoints the First Lord of the Admiralty, the Speaker of the House of Commons, the First Commissioner of Trade, the Admirals of the Red, White, and Blue Squadrons, the Master of Trinity-House, the President of the Royal Society, the Royal Astronomer at Greenwich, the two Savilian Professors at Oxford, and the Lucasian and Plumian Professors at Cambridge, with several other persons, as Commissioners for the Longitude at Sea. The Lowndian Professor at Cambridge was afterwards added. After this Act of Parliament, several other Acts passed, in the reigns of *George* II. and III, for the encouragement of finding the longitude. At last, in the year 1774, an act passed, repealing all other Acts, and offering separate rewards to any person who shall discover the longitude, either by the lunar method, or by a watch keeping true time, within certain limits, or by any other method. The

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* See *Whiston's* account of the proceedings to obtain this Act, in the Preface to his *Longitude discovered by Jupiter's Planets.*

Act proposes, as a reward for a time-keeper, the sum of 5000*l.* if it determine the longitude to 1°, or 60 geographical miles; the sum of 7500*l.* if it determine the same to 40 miles; and the sum of 10000*l.* if it determine the same to 30 miles, after proper trials specified in the act. If the method be by improved solar and lunar Tables, constructed upon Sir *I. Newton*'s theory of gravitation, the author shall be entitled to 5000*l.* if such Tables shall show the distance of the moon from the sun and stars within 15″ of a degree, answering to about 7′ of longitude, after making an allowance of half a degree for the errors of observation. And for any other method, the same rewards are offered as those for the time-keeper, provided it gives the longitude true within the same limits, and be practicable at sea. The commissioners have also a power of giving smaller rewards, as they shall judge proper, to any one who shall make any discovery for finding the longitude at sea, though not within the above limits. Provided, however, that if such person or persons shall afterwards make any further discovery so as to come within the above-mentioned limits, such sum or sums, as they may have received, shall be considered as part of such greater reward, and deducted therefrom accordingly.

(379.) After the decease of Mr. *Flamstead*, Dr. *Halley*, who was appointed to succeed him, made a series of observations on the moon's transit over the meridian, for a complete revolution of the moon's apogee, which observations being computed from the Tables then extant, he was enabled to correct the Tables of the moon's motion. And as Mr. *Halley* had then invented an instrument by which altitudes could be taken at sea, and also the moon's distance from the sun or a fixed star, Dr. *Halley* strongly recommended the method of finding the longitude from such observations, having found, from experience, the impracticability of all other methods, particularly at sea.

To find the Longitude by the Moon's Distance from the Sun, or a fixed Star.

(380.) The steps by which the longitude is found by this method, are these:

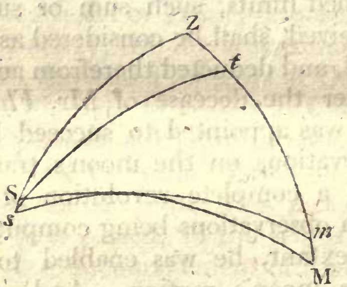
From the *observed* altitudes of the moon and the sun, or a fixed star, and their *observed* distance, compute the moon's *true* distance from the sun or star.

From the *Nautical Almanac*, find the time at Greenwich when the moon was at that distance.

From the altitude of the sun or star, find the time at the place of observation.

The difference of the times thus found, gives the difference of the longitudes.

(381.) To find the *true* distance of the moon from the sun or star by observation, let Z be the zenith, S the apparent place of the sun or a star, s the true place, M the apparent place of the moon, m it's true place; then the altitudes of M and s being known, by observation, the refractions Ss, Mm are known; also MS is known by observation; hence, in the triangle ZSM, we know SM the apparent distance, SZ, ZM, the complements of the apparent altitudes, to find the angle Z (Trig. Art. 239); and then in the triangle sZm, we know the angle Z, and sZ, mZ, the comple-



ments of the true altitudes, to find *sm* the *true* distance* (Trig. Art. 233).

Ex. Suppose on June 29, 1793, the sun's apparent zenith distance *ZS* was observed to be 70°. 56′. 24″, the moon's apparent zenith distance *ZM* to be 48°. 53′. 58″, their apparent distance *SM* to be 103°. 29′. 27″, and the moon's horizontal parallax to be 58′. 35″; to find their *true* distance *sm*.

The *true* distance *sm*, computed by the above method, is 103°. 3′. 18″.

(382.) The *true* distance of the moon from the sun or star being thus found, we are next to find the time at Greenwich. For this purpose, the sun or such fixed stars are chosen, as lie in or very near the moon's way, so that, looking upon the moon's motion to be uniform for a small time, the moon may be considered as approaching to, or receding from, the sun or star uniformly. To determine, therefore, the time at Greenwich corresponding to any given true distance of the moon from the sun or star, the *true* distance is computed in the *Nautical Almanac* for every three hours, for the meridian of Greenwich. Hence, considering that distance as varying uniformly, the time corresponding to any other *true* distance may be thus computed: Look into the *Nautical Almanac*, and take out two distances, one next greater and the other next less than the *true* distance deduced from observation, and the difference *D* of these distances gives the access of the moon to, or recess from, the sun or star in three hours; then take the difference *d* between the moon's distance at the beginning of that interval and the distance deduced from observation, and then say, *D* : *d* :: 3 hours : the time the moon is acceding to, or receding from, the sun or star by the quantity *d*; which added to the time at the beginning of the interval, gives the apparent time at Greenwich, corresponding to the given true distance of the moon from the sun or star.

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* There are shorter methods than this direct one, of computing the true distance, as the reader will see in my *Complete System of Astronomy*; but we here purpose only to explain the principles by which the longitude may be thus found.

Ex. On June 29, 1793, in latitude 52°. 12′. 35″, the sun's altitude in the morning was, by observation, 19°. 3′. 36″, the moon's altitude was observed to be 41°. 6′. 2″, the sun's declination at that time was 23°. 14′. 4″, and the moon's horizontal parallax 58′. 35″; to find the apparent time at Greenwich.

True dist. of ☾ from ⊙ by Art. 381. - 103°. 3′. 18″
True dist. by *Naut. Alm.* on June 29, at 3*h.* 103. 4. 58
True dist. by *Naut. Alm.* on June 29, at 6*h.* 101. 26. 42

d = ... 0. 1. 40
D = ... 1. 38. 16

Hence, 1°. 38′. 16″ : 0°. 1′. 40″ :: 3*h.* : 0^{h}. 3′. 3″, which added to 3*h.* gives 3*h.* 3′. 3″, the apparent time at Greenwich, when the *true* distance was 103°. 3′. 18″.

(383.) Find the apparent time at the place of observation, by the altitude of the sun (12). Then the difference of the times at Greenwich, and at the place of observation, is the distance of the meridians in time.

(384.) Now to find the apparent time at the place of observation, we have the sun's declination 23°. 14′. 4″, it's altitude 19°. 3′. 36″, it's refraction 2′. 44″, and parallax 8″; hence, it's true altitude was 19°. 1′, and therefore it's true zenith distance was 70°. 59′; also, the complement of declination was 66°. 45′. 36″; hence, by Art. 92:

66°. 46′. 56″ - - ar. co. sin. 0,0367325
37. 47. 25 - - ar. co. sin. 0,2127004
70. 56. 24

175. 29. 45

87. 44. 52 - - - - sin. 9,9996644
16. 48. 28 - - - - - sin. 9,4601408

2)19,7092381

9,8546190

the cosine of 44°. 18′. 52″, which doubled gives 88°. 37′. 44″, the hour-angle from apparent noon, which in time gives 5*h*. 54′. 31″, the time before apparent noon, or 18*h*. 5′. 29″, on June 28. Hence,

Apparent time at place of observ.	June 28,	18^h. 5′. 29″
———— at Greenwich,	June 29,	3. 3. 3
Difference of meridians in time - - -		8. 57. 34

Which converted into degrees, gives 134°. 23′. 30″, the longitude of the place of observation west of Greenwich.

To find the Longitude by a Time-keeper.

(385.) Let the *time-keeper* be well regulated, and set to the meridian of Greenwich; then if it neither gain nor lose, it will always show the time at Greenwich. Hence, to find the longitude of any other place, find the mean time from the sun's altitude by Art. 92; and observe, at the instant of taking the altitude, the time by the watch; and the difference of these times, converted into degrees, at the rate of 15° for an hour, gives the longitude from Greenwich. If, for example, the time by the watch, when the altitude was taken, was 6*h*. 19′, and the mean time deduced from that altitude was 9*h*. 23′, the difference 3*h*. 4′, converted into degrees, gives 46° the longitude of the place *east* from Greenwich, because the time at the place of observation is *forwarder* than that at Greenwich. Thus the longitude could be very readily determined, if you could depend upon the watch. But as a watch will always gain or lose, before it is sent out, it's gaining or losing every day for some time, a month for instance, is observed; this is called the *rate of going* of the watch, and from thence the *mean* rate of going is thus found.

(386.) Suppose, for instance, I examine the rate of a watch for 30 days; on some of those days I find it has gained, and on some it has lost; add together all the quantities which it has gained, and suppose they amount to 17″; add together all the quantities which it has lost, and let the sum be 13″; then the difference 4″ is the *mean* rate of gaining for 30 days, which divided by 30, gives 0″,133 for a *mean daily rate* of gaining. Or you may get the mean daily rate thus. Take the *difference* between what the watch was too fast, or too slow, on the first and last days of observation, if it be too fast or too slow on each day; but take the *sum*, if it be too fast on one day and too slow on the other, and divide by the number of days between the observations*. And to find the time at the place of trial at any future period by this watch, you must put down, at the end of the trial, how much the watch is too fast or too slow; then subtract from the time shown by the watch, 0″,133 × number of days from the end of the trial, being the exact quantity which it has gained according to the above mean rate of gaining, and you are then supposed to get the true time affected with the error at the end of the trial. This would be all the error, if the watch had continued to gain according to the above rate; and although, from the different temperatures of the air to which the watch may be exposed, and from the imperfection of the workmanship, this cannot be expected, yet, by taking it into consideration, the probable error of the time will be diminished. In watches which are under trial at the Royal Observatory at Greenwich, as candidates for the rewards offered by Parliament for the discovery of the longitude, this allowance of a mean rate, to be applied in order to get the time, is not granted by the Act of Parliament, but it requires that the watch itself should go within the limits assigned; the Commissioners, however, are so indulgent as to grant the application of a mean rate, which is undoubtedly favourable to the watches.

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* For further information on this subject, see Mr. WALES's *Method of finding the Longitude at Sea.*

(387.) As the rate of going of a watch is subject to vary from so many circumstances, the observer, whenever he goes ashore and has sufficient time, should compare his watch, for several days, with the mean time deduced from the altitude of the sun or a star, by which he will be able to determine it's rate of going. And whenever he comes to a place whose longitude is known, he may correct his watch, and set it to Greenwich time. For instance, if he go to a place known to be 30° east longitude from Greenwich, his watch should be two hours slower than the time at that place. Find, therefore, the time at that place by the altitude of the sun or a fixed star, and correct it by the equation of time, and compare the time so found with the time by the watch when the altitude was taken, and if the watch be two hours slower than the time deduced from observation, it is right; if not, correct it by the difference, and it again gives Greenwich time.

(388.) In long voyages, unless you have sometimes the means of adjusting the watch to Greenwich time, it's error will probably be very considerable, and consequently the longitude deduced from it will be subject to a proportional error. In short voyages, a watch is undoubtedly very useful, and also in long ones, where you have the means of correcting it from time to time. It serves to carry on the longitude from one known place to another, supposing the interval of time not to be very long; or to keep the longitude from that which is deduced from a lunar observation, till you can get another observation. Thus the watch may be rendered of great service in Navigation.

To find the Longitude by an Eclipse of the Moon, and of Jupiter's Satellites.

(389.) By an eclipse of the moon. This eclipse begins when the umbra of the earth first touches th moon, and it ends when it leaves the moon. Having the times calculated when the eclipse begins and ends at Greenwich, observe the times when it begins and ends at any other place; the difference of these times converted into degrees, gives the difference of longitudes. For as the phases of the moon in an eclipse happen at the same instant to every observer, the difference of the times at different places, when any phase is observed, will give the difference of the longitudes. This would be a very ready and accurate method, if the time of the first and last contact could be accurately observed; but the darkness of the penumbra continues to increase till it comes to the umbra, so that, until the umbra actually gets upon the moon, it is not discovered. The umbra itself is also very badly defined. The beginning and end of a lunar eclipse cannot, in general, be determined nearer than 1′ of time; and very often not nearer than 2′ or 3′. Upon these accounts, the longitude, from the observed beginning and end of an eclipse, is subject to a considerable degree of uncertainty. Astronomers, therefore, determine the difference of the longitudes of two places by corresponding observations of other phases, that is, when the umbra bisects any of the spots upon the moon's surface. And this can be determined with a greater degree of accuracy than the beginning and end; because, when the umbra is gotten upon the moon's surface, the observer has leisure to consider and fix upon the proper line of termination, in which he will be assisted by running his eye along the circumference of the umbra. Thus the coincidence of the umbra with the spots may be observed with tolerable accuracy. The observer, therefore, should have a good map of the moon at hand, that he may not mistake. The telescope, to observe a lunar eclipse, should have but a small magnifying power with a great deal of light. The shadow comes upon the moon on the east side, and goes off on the west; but if the telescope invert, the appearances will be contrary.

(390.) The eclipses of *Jupiter*'s satellites afford the readiest method of determining the longitude of places at land. It was also hoped that some method might be invented to observe them at sea, and Mr. *Irwin* made a chair to swing for that purpose, for the observer to sit in; but Dr. *Maskelyne*, in a voyage to Barbadoes, under the direction of the Commissioners of Longitude, found it totally impracticable to derive any advantage from it; and he observes, that, "considering the great power requisite in a telescope for making these observations well, and the violence as well as irregularities of the motion of a ship, I am afraid the complete management of a telescope on ship-board will always remain among the desiderata. However, I would not be understood to mean to discourage any attempt, founded upon good principles, to get over this difficulty." The telescopes proper for making these observations, are common refracting ones from 15 to 20 feet, reflecting ones of 18 inches or 2 feet, or the 46-inch achromatic with three object glasses, which were first made by Mr. *Dolland*. On account of the uncertainty of the theory of the satellites, the observer must be settled at his telescope a few minutes before the expected time of an immersion; and if the longitude of the place be also uncertain, he must look out proportionably sooner. Thus, if the longitude be uncertain to 2°, answering to eight minutes of time, he must begin to look out eight minutes sooner than is mentioned above. However, when he has observed one eclipse, and found the error of the Tables, he may allow the same correction to the calculations of the Ephemeris for several months, which will advertise him very nearly of the time of expecting the eclipses of the same satellite, and dispense with his attending so long. Before the opposition of Jupiter to the sun, the immersions and emersions happen on the west side of Jupiter, and after opposition, on the east side; but if the telescope invert, the appearance will be the contrary. Before opposition, the immersions only of the first satellite are visible; and after opposition, the emersions only. The same is generally the case with respect to the second satellite; but both immersion and emersion are frequently observed in the two outer satellites.

(391.) When the observer is waiting for an emersion, as soon as he suspects that he sees it, he should look at his watch, and note the second, or begin to count the beats of the clock, till he is sure that it is the satellite, and then look at the clock, and subtract the number of seconds which he has counted from the time then observed, and he will have the time of emersion. If Jupiter be 8° above the horizon, and the sun as much below, an eclipse will be visible; this may be determined near enough by a common globe.

(392.) The immersion or emersion of a satellite being observed according to apparent time, the longitude of the place from Greenwich is found, by taking the difference between that time and the time set down in the *Nautical Almanac*, which is calculated for apparent time.

Ex. Suppose the emersion of a satellite to have been observed at the Cape of Good Hope, May 9, 1767, at 10*h*. 46′. 45″ apparent time; now the time in the *Nautical Almanac* is 9*h*. 33′. 12″; the difference of which time is 1*h*. 13′. 33″, the longitude of the Cape *east* of Greenwich in time, or 18°. 23′. 15″.

(393.) But to find the longitude of a place from an observation of an eclipse of a satellite, it is better to compare it with an observation made under some well-known meridian, than with the calculations in the Ephemeris, because of the imperfection of the theory; but where a corresponding observation cannot be obtained, find what correction the calculations of the Ephemeris require, by the nearest observations to the given time that can be obtained; and this correction, applied to the calculation of the given eclipse in the Ephemeris, renders it almost equivalent to an actual observation. The observer must be careful to regulate his clock or watch by apparent time, or at least to know the difference; this may be done, either by equal altitudes of the sun, or of proper stars; or the latitude being known, from one altitude at a distance from the meridian, the time may be found by Art. 92.

(394.) In order the better to determine the difference of longitudes of two places from corresponding observations, the observers should be furnished with the same kind of telescopes. For at an immersion, as the satellite enters the shadow, it grows fainter and fainter, till at last the quantity of light is so small that it becomes invisible, even before it is immersed in the shadow; the instant, therefore, that it becomes invisible will depend upon the quantity of light which the telescope receives, and it's magnifying power. The instant, therefore, of the disappearance of a satellite will be later the better the telescope is, and the sooner it will appear at it's emersion. Now the immersion is the instant the satellite is wholly gotten into the shadow, and the emersion is the instant before it begins to emerge from the shadow; if, therefore, two telescopes show the disappearance or appearance of the satellite at the same distance of time from the immersion or emersion, the difference of the times will be the same as the difference of the true times of their immersions and emersions, and therefore will show the difference of longitudes accurately. But if the observed time at one place be compared with the computed time at another, then we must allow for the difference between the apparent and true times of immersion or emersion, in order to get the true time where the observation was made, to compare with the true time from computation at the other place. This difference may be found, by observing an eclipse at any place whose longitude is known, and comparing it with the time by computation. Observers, therefore, should settle the difference accurately by the mean of a great number of observations thus compared with the computation, by which means the longitude will be ascertained to a much greater accuracy and certainty. After all this precaution, however, the different states of the air at different times, and also the different states of the eye, will introduce a small degree of uncertainty; the latter case may perhaps, in a great maasure, be obviated, if the observer will be careful to remove himself from all warmth and light for a little time before he makes the observation, that the eye may be reduced to a proper state; which precaution the observer should also attend to, when he settles the difference between the apparent and true times of immersion and emersion. Perhaps also the difference arising from the different states of the air might, by proper observations, be ascertained to a considerable degree of accuracy; and as this method of determining the longitude is, of all others, the most ready, no means ought to be left untried to reduce it to the greatest certainty.

TABLE I.

For converting Degrees, Minutes, and Seconds, into Time, at the Rate of 360 Degrees for 24 Hours.

Deg. Min.	*Hou. Min.*	*Min. Sec.*	*Deg. Min.*	*Hou. Min.*	*Min. Sec.*	*Sec.*	*Dec. of Sec.*
1	0	4	30	2	0	1	,066
2	0	8	40	2	40	2	,133
3	0	12	50	3	20	3	,2
4	0	16	60	4	0	4	,266
5	0	20	70	4	40	5	,333
6	0	24	80	5	20	6	,4
7	0	28	90	6	0	7	,466
8	0	32	100	6	40	8	,533
9	0	36	200	13	20	9	,6
10	0	40	300	20	0	10	,666
20	1	20					

TABLE II.

For converting Time into Degrees, Minutes, and Seconds, at the Rate of 24 Hours for 360 Degrees.

Hou.	*Deg.*	*Min. Sec.*	*Deg. Min.*	*Min. Sec.*	*Dec. of Sec.*	*Sec.*
1	15	1	0	15	,1	1,5
2	30	2	0	30	,2	3,0
3	45	3	0	45	,3	4,5
4	60	4	1	0	,4	6,0
5	75	5	1	15	,5	7,5
6	90	6	1	30	,6	9,0
7	105	7	1	45	,7	10,5
8	120	8	2	0	,8	12,0
9	135	9	2	15	,9	13,5
10	150	10	2	30		
11	165	20	5	0		
12	180	30	7	30		
16	240	40	10	0		
20	300	50	12	30		

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